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Forestry Trophodynamics in Deformed Media Allometric Spaces and Carbon Flux in the Understory Rhizosphere

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We have shown elsewhere that the classical constant coefficient 2-species ecological population *density*, $N^i(t)$, $i = 1, 2$, interaction equations for parasitism, mutualism and competition[4], when augmented with the Volterra production equation,

$$x^i(t) = k_{(i)} \int_0^t N^i(s)\, ds + x^i(0),$$

conserve Medawar (Finslerian) growth energy along solution curves in production space viewed as a continuous medium subject to elastic and plastic deformation analogous to continumm mechanics. Thus, compatibility of deformations (zero curvature) $dX^i = B^i_r dx^r$ must be exhibited to ensure a local disturbance can prolongate to a global deformation. The plant species production is *plastically deformed*[5] from the original three energy efficient, *elastically equivalent*[6] production processes, each conforming to Huxley's allometric law. In this perspective, the original Cartesian Huxley space is upgraded to a set of 3 each with a unique Finslerian metric and geodesics elastically equivalent to the original *ideal* Cartesian one. These are called *perfect.* The set as a whole is refered to as the *Finsler Gate and denoted* A_α. We have proved elsewhere that the 3 classical ecological interactions, when viewed as production processes, are plastic deformations of one of the 3 perfect production processes and that there are exactly 8, second-order ordinary differential equation constant coefficient systems (SODE's), 3 being elastic with coefficients denoted, A^i_{jk}, and 5 (being plastic deformations of euclidean Huxley space) with constant coefficients denoted, $A^i_{jk} + W^i_{jk}$, and all 8 conserving Medawar growth energy. Moreover, the 5 plastic ones

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[4]Competition requires each species have a refuge from the other in order that conservation occurs.

[5]Plastic deformation is a *non-integrable* change of coordinates and uses *non-holonomic frames* for their description.

[6]I.e., non-singular coordinate change viewed as a *motion* in the original Cartesian Huxley space) production processes, each of which conforms to Huxley's allometric law.

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are curved when expressed in terms autoparallels of a constant Wagner (asymmetric) connection, $W^i_{jk} = \delta^i_j \sigma_k$, $\partial_k \sigma(x) = \sigma_k$, but are not geodesics. This connection has vanishing curvature so the associated deformation proagates globally. Furthermore, an ***evolutionary process*** called ***heterochrony*** enforces **a time-sequencing change along growth and developmental trajectories** that eventually, **via Darwinian selection**, cause the SODE to be **optimally energy efficient**, or nearly so. For example, production for competition between 2 tree species, say, Fir and Birch in the Canadian Northwest, growing at the same rate λ, will evolve to become geodesics through deep time. The equations (using Einstein summation convention) are

$$dy^i/dp + \{A^i_{jk} + W^i_{jk}\}y^j y^k = F^i_j y^j,\ F^i_j = g^{il}F_{lj},\ F_{lj} = \delta_l A_j - \delta_j A_l,$$
$$A_l = \sigma y_l,\ y^i = dx^i/ds,\ ds/dt = \mathrm{Exp}\{\lambda t\},$$
$$dp/ds = \mathrm{Exp}\{ -\int_\gamma \sigma(r)\ dr \},\ \delta_i = \partial_i - A^q_i \dot{\partial}_q,\ A^q_{im} y^m = A^q_i,$$

with $g_{rs} = (1/2)\dot{\partial}_r \dot{\partial}_s \big[A_\alpha(y)\big]^2$, $\alpha = 1, 2, 3$, index α, indicates which metric in the Finsler Gate is used.

(The dot over the partial indicates partial derivation with respect to y and γ denotes a solution curve.)

The example of Fir and Birch and the model equations given above do not explain explain how soil carbon is peridically cycled between the two over a season. There is a missing ingredient, namely, the *rhizosphere* consisting of *mycorrhizal fungus* surrounding the roots of all plants nearby. This is an extensive underground network for transporting water and trading carbon, nitrogen and phosphorous and other nutrients essential for growth, respiration and maintenance of these forest dwellers. We wish to extend the *elastic/plastic deformation method* to cover a more realist situation: 2 species embedded in a larger community of understory plants and also their fungal symbiontsin the underground rhizosphere. By adding $b_l(x)$ to A_l we obtain a system that exhibits vortex motion in production space provided N^1/N^2 is constant in some season or part thereof. So choosing $\alpha = 2$ we obtain a model of the Fir/birch carbon fluxuations as discovered in the last few years. The model is similar to the Lorentz equations for a charged particle in and electromagnetic field. In fact, the frame theory we use is a correction and generalization of the frame discovered by P. Holland. However, the fact that adding a vector field to the metric of an already deformed medium yields another for the composite anholonomic frame is an exercise.

Referências

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