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Synchronization of Stuart-Laudau oscillators' networks with randomly distributed coupling constants

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Spread through areas such as physics and biology, the synchronization phenomena is frequently observed both in nature and manmade systems, with some of its most classical sightings being the neuronal rithm on mammals, power grids, the collective flashing of fireflies, chemical oscillators, the pacemaker cells in our hearts, electrons in a superconductor, and many others [2, 4].

For a network with oscillators as nodes coupled with each other through some given topological pattern, global synchronization can be understood as the phenomena for which after some time all oscillators tend to adopt the same common frequency Ω, creating a phase transition for the whole system.

The most famous mathematical model developed for the study of networks' synchronization is the *Kuramoto model* [1], proposed by Yoshiki Kuramoto in 1975 and widely used to this day for its eficcient treatment of large oscillator populations by employing typical tools of statistical physics. For a network of Stuart-Landau (SL) oscillators, which describes the general oscillatory behavior near a Hopf bifurcation, the Kuramoto model can be adapted so that the whole system is represented by the following dynamics [3]

$$\dot{z}_k = (\alpha^2 + i\omega_k - |z_k|^2)z_k + \sum_{j=1}^{N} \lambda_{ij} A_{kj}(z_j - z_k) \tag{1}$$

where $z_k = \rho_k e^{i\theta_k}$ is the complex state of the oscillator correspondent to node k, ω_k its natural frequency, and α is a control parameter which determines the stability properties of the limit cycle $|z_k|^2 = \alpha^2$ of the isolated SL oscillator ($\lambda_{kj} = 0$). The coupling is governed by both the Adjacency matrix A_{kj} and the varying coupling strength λ_{kj}.

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One can quantify the phase synchronizing behaviour of a system by its so called *order parameter* r, whose definition reads

$$r(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_k} \right| \tag{2}$$

with $r \to 1$ as the system becomes globally synchronized. It's worth noticing that even though several properties of such systems can be derived analitically, a great share of results still depend on numerical simulations, and therefore the computational side of this problem should also be explored thoughout this work.

Research around the synchronization properties have been extensively done in the past few years for networks with somewhat standard topologies, in the sense of having the coupling constantly assumed to be either the same for every node or a function of systems parameters such as the natural frequencies. We, however, turn our attention towards how random distributions for the coupling strengths λ_{jk} may affect the synchronization process, seeking relationships between the distribution parameters such as width or standard deviation and the synchronization properties of the SL network system.

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