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# Metaheuristics for Computing Cosmic Microwave Background Maps

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**Abstract**.

Temperature differences data sets obtained from a telescope can be transformed into a Cosmic Microwave Background (CMB) map. Our formulation for addressing the CMB mapping is based on an inverse problem methodology minimizing an optimization problem. Noisy synthetic date representing the BEAST experiment 30-day mission were used to test our approach. The computed CMB maps) are similar to those obtained by the bin average method. Our approach produced better results in similar processing time.

**Key words**. CMB maps, inverse problems, simulated annealing, genetic algorithms.

## 1 Introduction

Cosmic Microwave Background radiation (CMB) maps are important tools to compute temperature power spectra, allowing to compute cosmological parameters used to check the standard cosmological model [17]. CMB maps are constructed by measuring noisy sky temperatures differences in microwave frequencies, estimating the corresponding noiseless temperatures and plotting the temperature anisotropies [12]. Such temperature anisotropy is the key to explain the Universe structure on large scale [9]. This was done, for instance, by the Cosmic Background Explorer satellite (COBE) team, measuring the CBM spectrum [13]. The CMB map making process consists in reducing a large time-ordered data set of temperatures acquired by a telescope into a map of sky temperature anisotropies of the observed region of the sky [2].

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A pointing matrix correlates the temperature measurements to the observed regions in the sky, being defined by the pointing strategy of the instrument. The map-making process constitutes the inverse problem of estimating temperatures anisotropy values from temporal ordered data (TOD) of noisy temperature measurements. The COBE team developed the standard inverse method based upon a least squares approach, which leads to a system of linear equations [19]. A very common approach to reduce the high computational complexity of this method is the bin average method, that averages the temperatures successively acquired for each pixel.

Here, the estimation of the CMB maps is computed as an inverse problem. The inverse problem is formulated as an optimization problem. Two metaheuristics were employed: genetic algorithm and simulated annealing. Both techniques, implicit inverse problem and bin average, are evaluated with simulated data from the Background Emission Anisotropy Scanning Telescope (BEAST) experiment [14,16].

# 2 Making CMB Maps through Inverse Problem Approach

The CMB map-making process considered here is formulated as an implicit inverse problem. The cost function is the square difference between the set of the estimated temperatures ($T_e$) and the set of measured temperatures ($T_d$) of the sky:

$$F^k = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ T_{e_i}^k - T_{d_{ij}} \right]^2 \ . \tag{1}$$

In this inverse formulation, the *cause* is the set of temperature field $T_e$ and the *effect* is the set of measured temperatures ($T_d$). The square norm between estimated and observed temperatures is expressed by pixelization scheme. Since the measured temperatures are independent for each pixel, each temperature is independently estimated The estimated temperature Te for the BEAST experiment is computed considering a pointing matrix expressed by: $P_{ij} = \delta_{P(i,j)}$ (Kroeneker delta function).

## 2.1 Solving the Optimization Problem by a Genetic Algorithm

Genetic Algorithms (GA) are stochastic search methods based on a population evolution under natural selection. Each individual has its particular genotype and is associated to a candidate solution. New individuals in the next generation are generated using operators for selection, crossover, and mutation. Particularly, real-coded GA's have been employed in numerical optimization with good results [15].

Our individuals are sets of temperatures, coded by real numbers, being each gene associated to a particular pixel temperature. A standard GA [7] was implemented. Selection is randomly performed by means of a roulette wheel. Single point crossover is adopted and individual genes can undergo mutation with a probability of 1%. A second GA (denoted as *Boltzmann*) was also implemented using the Boltzmann distribution to evaluate the probability of selection for each individual, in place of the uniform distribution of the canonical GA. In addition, it employs a two-point crossover, and mutation with a probability of 0.5% applied only to genes that did not undergo crossover.

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### 2.2 Solving the Optimization Problem by Simulated Annealing

Simulate Annealing (SA) is a metaheuristic for optimization problems [11]. The SA algorithm associates an amount of energy $E$ to each possible state $S$ of the system, associated to a point of the search space of the problem. The energy is calculated according to the specific optimization goal of the problem. The algorithm tries to reach a state of minimum energy energy = objective function). The SA starts at an arbitrary state and a random new state is addressed at every iteration step, according to a certain probability. This probability is a function of the energy difference between the two states and a global step-dependent parameter defined as the *temperature* $T_{\rm SA}$ of the system. If the energy of the new state is lower than the previous value, the transition to the new state is accepted. If it is higher, it might be accepted or not, according to a random number that is compared to the value $p$ of a probability distribution function (PDF) that depends on the SA temperature. If the random number is greater than $p$, the new state is accepted. The standard SA adopts the Boltzmann probability distribution function and the annealing schedule is logarithmic: on the iteration step $w$, the temperature is given by $T^w = T^0/\log(w)$. The value of $p$ is given by (below, $k_B$ is the Boltzmann constant):

$$p = \exp\left[-\Delta E/(k_B T_{\rm SA})\right] \; . \tag{2}$$

The scheme known as Fast Simulated Annealing (FSA) employs the Cauchy-Lorentz distribution [18], while the Adaptive Simulated Annealing (ASA) employs a Gaussian distribution [10]. Here, the FSA adopts a linear annealing schedule ($T_w = T^0/w$), while ASA employs: $T^w = T^0 \exp(-c_w w^{1/D})$ [10], where $c_w$ is a constant, and $D$ is the total number of parameters. SA algorithms may be improved by including a re-annealing phase. This phase occurs when the evaluation of the objective function remains unchanged for a chosen number of iterations. As the algorithm stabilizes, i.e. converges to an optimal solution, a perturbation is applied to the solution, yielding a new solution $T_e^\eta$.

A new scheme to calculate the perturbed solution for the re-annealing is proposed, employing average/mean absolute deviation $\alpha$ of the last candidate solution that is calculated from the mean absolute deviation of all its $n$ pixels, as follows [20]:

$$\alpha = \frac{1}{n}\sum_{i=1}^{n} |T_{e_i} - \bar{T}_i| \; , \qquad \text{where: } \bar{T}_i = \langle T_i \rangle_i \; . \tag{3}$$

The mean temperature of the measurements is denoted by $T_i$ , and $T_{e_i}$ represents the value of the last candidate solution, both for pixel $i$. The perturbed solution $T_e^\eta$ then uses this mean absolute deviation $\alpha$ as a perturbation factor to all the temperatures of this solution. This factor is weighted by an empirical factor $\xi(= 0.4)$ for all the $n$ pixels, as follows:

$$T_{e_i}^\eta = T_{e_i}\,(1 - \alpha\xi) \; . \tag{4}$$

## 3 Results with Synthetic Observations

Simulated observation data emulates a 30-day mission of the BEAST experiment. A full sky map was created using the SYNFAST routine of the HEALPIX package [8]. A smaller 8000- pixel patch of this sky map that corresponds to a particular one-hour daily acquisition of the BEAST experiment was selected for the numerical tests, and corrupted with white/Gaussian noise in a way to obtain a 0.1 signal-to-noise ratio. Each one of the simulated TOD series is composed by 20,000 temperature values for each pixel.

Results evaluation is performed by means of the Pearson correlation coefficient $\rho$ and the mean absolute deviation $\alpha$, both taken between the exact and the estimated maps. Reference values are calculated from the bin average. One-hour sets of data are pixelwise averaged, instead of performing the average of all measurements at a time. Next, the hourly averages are averaged for each week and, finally, the week averages are averaged for an entire year. The resulting vector is a CMB map. These averages must be weighted by the standard deviation of the $m$ measurements, for each pixel of a particular TOD.

## 3.1 Inverse solution by Genetic Algorithm

Inverse solution computed using GAs was obtained with 100 individuals in the population, and each individual was composed by 8,000 genes, one per pixel. Each gene is real coded in such a way that allows a range 0f -999 $\mu$K to 999 $\mu$K in the temperature anisotropy. Figure 1 shows the results for the three types of GAs: the canonical, the global Boltzmann, and the island Boltzmann. The better performance was obtained with the global Boltzmann GA. Although the global Boltzmann GA presented the best quality map among the GAs, this quality (evaluated by the Pearson correlation coefficient $\rho$) was worse than the map generated by the bin average method (reference), and the GA execution time was significantly higher (CPU-time for global Boltzmann GA = 1021 min, while CPU-time for bin average = 110 min).

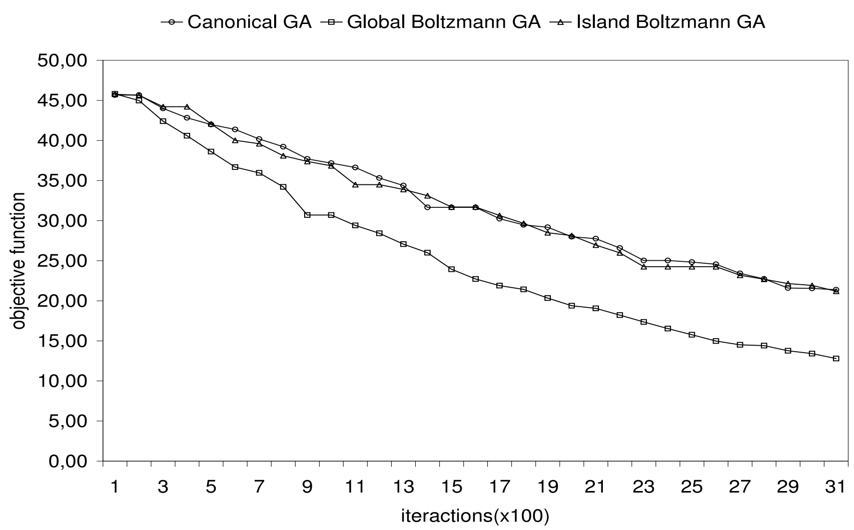


Figura 1: Convergence of the parallel GAs: objective function vs number of iterations.

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### 3.2 Inverse Solution by Simulated Annealing

Some SA with different PDFs (uniform, Boltzmann, and Cauchy-Lorentz) were tested for map making The convergence of these solutions is shown in Figure 2. The quality of these solutions is shown in the first three lines of Table 1. It can be noted that any SA yielded a better solution than the bin average method ($\rho = 0.999$ for SA algorithms, and $\rho = 0.995$ for bin average approach). However, the corresponding parallel processing times were about 8 times greater than the sequential time of the bin average method. The best SA, using the Cauchy-Lorentz PDF, was then adopted for further study (re-annealing).

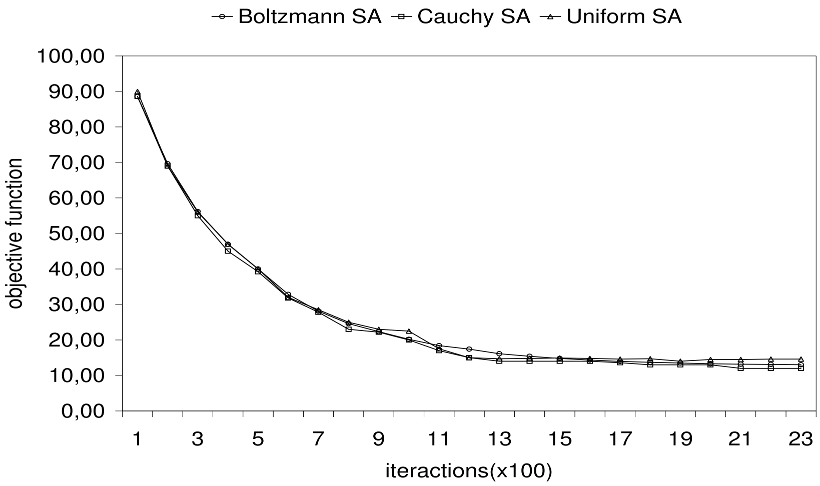


Figura 2: Convergence for SA implementations: objective function vs number of iterations.

Re-annealing approach was applied to the Cauchy-Lorentz SA with sampled initial guess (it is is employed if the value of the objective function did not decrease after 10 iterations), obtaining the the best quality solution obtained by SA implementations.

Tabela 1: Performance of the SA implementations and the bin average method: mean deviation ($\alpha$), Pearson correlation coefficient ($\rho$), final objective function value ($F_{\text{end}}$), total number of evaluations of this function ($N_{\text{end}}$), and minutes of processing time ($\Delta t$).

| Process | SA Algorihtm | $\alpha$ | $\rho$ | $F_{\text{end}}$ | $N_{\text{end}}$ | $\Delta t$ |
|---|---|---|---|---|---|---|
| PDF | Uniform | 0.290 | 0.999 | 19.47 | 1511 | 799 |
| PDF | Boltzmann | 0.285 | 0.999 | 18.51 | 1514 | 801 |
| PDF | Cauchy-Lorentz | 0.250 | 0.999 | 15.48 | 1497 | 792 |
| Re-anneling | Cauchy-Lorentz | 0.200 | 0.999 | 10.05 | 522 | 122 |
| Bin average | — | 0.290 | 0.995 | — | — | 110 |

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## 4 Final Remarks

A scheme for making Cosmic Microwave Background (CMB) maps is addressed based on inverse problem approach. The associated optimization problem is solved by using two metaheuriscs: Genetic Algorithm and Simulated Annealing. A comparison between the inverse solutions and the bin average solution is carried out. Noisy simulated data corresponding to the BEAST experiment were employed. The genetic algorithms presented a worse performance than bin average. All results with SA were better than bin average. The best result was obtained with re-annealing applied on Cauchy-Lorentz SA (Fast Simulated Annealing) with sampled initial guess.

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