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Comparison of discretization methods in linear hydrodynamic stability problems

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1 Introduction

The problem of analyzing the effects of perturbations over flows is known as a hydrodynamic stability problem. These problems appear in different scientific fields, and have been studied using a lot of different strategies. One of those is linear hydrodynamic stability analysis, where a perturbation is introduced in the flow equations and only the linear terms are considered. This strategy has been employed in different works such as [3], in terms of the stream function for the flow (particularly, the Poiseuille flow) and using Chebyshev collocation (a spectral method). The difficulties of reproducing the same steps arise in more complex geometries, since it's not possible to derive the stream function from the velocity field. Another is related to the discretization method employed, considering it's convergence only happens when the solution in the whole domain is smooth. The problem may also be described using the primitive variables (velocity and pressure), and using non global discretization methods such as finite differences or finite elements. This approach is more appropriate than the former considering more complex problems, at the cost of having larger matrices and the anomaly known as the infinite eigenvalues. The latter can be treated using the techniques discussed in [1] and [2].

2 Objective

In order to capture the similarities, differences and adversities on the ways of solving the linear stability for a flow, a comparison of distinct schemes is done. The following aspects will be analyzed: eigenvalues of the discrete system, matrix sizes, matrix structures and the eigensolvers. For the Couette flow, the spectrum and their respective matrices are shown in Figures 1 and 2.

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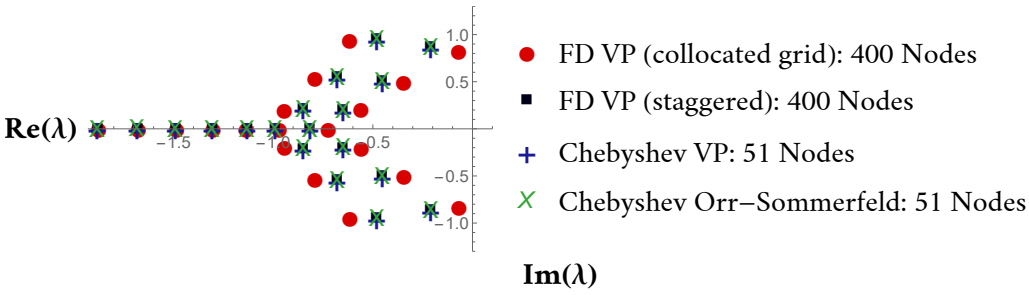
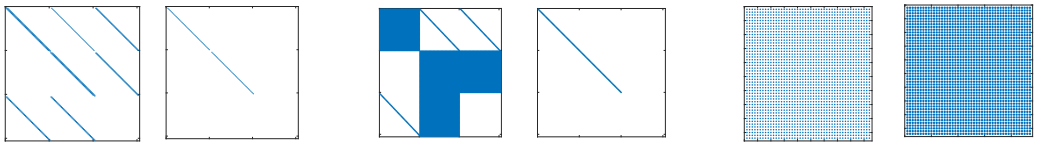


Figure 1: Couette flow spectrum with different discretizations. Different methods describe the same eigenvalues, but with observable differences.



(a) J VP, Finite Differences (b) M VP, Finite Differences (c) J VP, Chebyshev (d) M VP, Chebyshev (e) J Orr-Sommerfeld, Chebyshev (f) M Orr-Sommerfeld, Chebyshev

Figure 2: Matrices of the resulting generalized eigenvalue problem $J\mathbf{c} = \lambda M\mathbf{c}$ using different discretizations. The existence of blocks is present on the primitive variables formulation and matrices densities are associated with using a global or local discretization method.

By realizing this analysis in the Couette and other flows, this work aims to refine a scheme that discretizes linear stability problems in the most coeherent way and solve the associated generalized eigenvalue problem with ideas from [1] and [2].

References

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