Analysis of a 3-DOF shear building dynamics with nonlinear stiffness

José E. Montiel¹
Faculdade de Engenharia, Departamento de Engenharia Civil, UFGD, Dourados, MS

Marcus Varanis²
Faculdade de Engenharia, Departamento de Engenharia Mecânica, UFGD, Dourados, MS

This paper aims to model and analyze a three-story shear building (Figure 1) excited by an unbalanced motor in two cases, in order to identify the influence of non-linearity on such a system. In the first case, all the columns behave as linear springs whereas in the second case the second floor columns behave as Duffing nonlinear springs.

![Three-story shear building](image)

Figure 1: Three-story shear building

The parameters used are mass $m_i (i=1,2,3)$ of 0.416, 0.416 and 0.726 kg, stiffness of columns $k_i (i=1,2,3)$, all equal to 1.966 kN/m, and the motor frequency $\omega_3$ of 20 Hz. The unbalanced mass $m_u$ was 6.59 g and its distance $R$ from the axis of rotation was 15 mm. The coefficients related to the nonlinear spring are $\alpha = -1.966 \times 10^3$ and $\beta = 0.4915 \times 10^3$.

The motion equations are given by equations 1 and 2 and they have been solved by the fourth-order Runge-Kutta algorithm. The frequency domain responses (Figure 3) were obtained by using the Fast Fourier Transform (FFT).

\[
\begin{align*}
    m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) &= 0 \\
    m_2 \ddot{x}_2 + k_2 (x_2 - x_1) - k_3 (x_3 - x_2) &= 0 \\
    m_3 \ddot{x}_3 + k_3 (x_3 - x_2) &= m_u \omega_3^2 R \cos(\omega_3 t)
\end{align*}
\]

¹joseemontiel95@gmail.com
²MarcusVaranis@ufgd.edu.br
\[
\begin{align*}
    m_1 \ddot{x}_1 + k_1 x_1 - \alpha (x_2 - x_1) - \beta (x_2 - x_1)^3 &= 0 \\
    m_2 \ddot{x}_2 + \alpha (x_2 - x_1) + \beta (x_2 - x_1)^3 - k_3 (x_3 - x_2) &= 0 \\
    m_3 \ddot{x}_3 + k_3 (x_3 - x_2) &= m_u \omega_3^2 R \cos(\omega_3 t)
\end{align*}
\] (2)

Figure 2: Phase space of the third floor for each system

(a) Phase space of the linear system  (b) Phase space of the nonlinear system

Figure 3: Frequency domain responses obtained by using the FFT algorithm

(a) Linear system frequency domain response  (b) Nonlinear frequency domain response

These simulations showed the effects of nonlinearity on the characteristics of an oscillation. This can be seen in Figure 2 and also in Figure 3, where the appearance of the signal frequencies have changed drastically, indicating chaotic movement.

References
