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Inverse Modeling of Radiative Transfer by Two-Stream Approximation using the Luus-Jaakola Method.

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1 Introdution

The simulation of terrestrial ecosystem processes, using numerical biosphere-atmosphere models that can be directly coupled to the Atmospheric General Circulation Models (AGCM) and Regional Atmospheric Models, assists to a more accurate diagnosis and prognosis of climate and weather. These models describe the interaction processes with the atmosphere and the transient changes that occur in vegetation composition and structure.

The solar radiation module of the biosphere-atmosphere models is relevant when considering dynamic models due to being the main mechanism of energy contribution to an ecosystem. The distribution of solar radiation inside the vegetation canopy controls the entire dynamics of plant growth through the portion absorbed by photosynthesis. This also influences the assimilation of water and nutrients in the soil by the mechanisms of heat and mass exchange.

In this work the two-stream radiative transfer model, [2,5] was adopted. The Shooting Method was used to solve the Ordinary Differential Equations (ODE) system in the Scilab mathematical modeling environment, as well as the Luus-Jaakola method, [6], to obtain the necessary parameters to solve the direct problem.

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2 Mathematical Model

Consider the values obtained experimentally for the upward diffuse radiative flux. The residue between the calculated and the experimental quantity was given by

$$\mathbf{R} = S_c - S_e$$

where the subscript c indicates the value of S calculated computationally and the subscript e indicates the value of S experimentally obtained in the field, S (W/m^2). This method aimed to get a residue as small as possible. Hence, this was a minimization problem.

The functional of the square residuals was

$$Q(\mathbf{X}) = \frac{1}{2}\sum_{i=1}^{M}(S_{c_i} - S_{e_i})^2 \tag{1}$$

where $\mathbf{R} = (R_1, \ldots, R_M)^T \in I\!R^M$ represents the residue, M is the amount of experimental data, and $\mathbf{X}$ represents the parameters to be estimated, [8].

2.1 Upward and Downward diffuse radiative fluxes: Two-Stream

The model used in this work is based on the two-stream approximation method applied to the radiative transfer equation to obtain a two-fluxes models in vegetable canopies. All calculations were solved separately for two wavelengths: visible (0.4 to 0.7 μm) and near infrared (0.7 to 4.0 μm).

Within each layer, the upward and downward radiation fluxes [1, 4, 7, 9], respectively, are described by

$$-\bar{\mu}\frac{\mathrm{d}S^{\uparrow}}{\mathrm{d}L} + (1-(1-\beta)\omega)S^{\uparrow} - \omega\beta S^{\downarrow} = \omega\beta_0\bar{\mu}\frac{G}{\mu}D_0 e^{-GL/\mu} \tag{2a}$$

$$\bar{\mu}\frac{\mathrm{d}S^{\downarrow}}{\mathrm{d}L} + (1-(1-\beta)\omega)S^{\downarrow} - \omega\beta S^{\uparrow} = \omega(1-\beta_0)\bar{\mu}\frac{G}{\mu}D_0 e^{-GL/\mu} \tag{2b}$$

where, $S^{\uparrow}$ and $S^{\downarrow}$ are the upward and downward diffuse radiative fluxes per unit incident flux (W/m^2), respectively, L is the leaf area index (LAI = leaf area/ground area) plus the stem area index (SAI = stem area/ground area) increasing in the downward direction (m^2/m^2), $\bar{\mu}$ is the average inverse diffuse optical depth per unit of $\mathrm{d}L$ (m^2/m^2), μ is the cosine of the incident direct beam radiation (cosine of the zenith angle) and D_0 its intensity (W/m^2), $G(\mu)$ is the relative projected area of leaf and stem elements in the direction $\cos^{-1}\mu$ (m^2/m^2), ω is the scattering coefficient, and β and β_0 are the upscatter parameters for diffuse and direct beam radiation, respectively [7]. The intensity of direct beam radiation within the layer is given by $D_0 e^{-GL/\mu}$.

2.1.1 Boundary conditions

The required boundary conditions were, for the top of the canopy, where $L = 0$, the value of $S^{\downarrow}(0) = S_0^{\downarrow}$, is the downward diffuse solar radiation at the top of the canopy

obtained through an empirical model based on the atmospheric conditions which was optimized by the minimum squares method, see Figure 1. For the upward radiation, which occurs under the canopy, where $L = L_b = LAI + SAI$, the value of $S^{\uparrow}$ was given by the equation $S^{\uparrow}(L_b) = S^{\uparrow}_{L_b} = \alpha_D(\mu)D_0 e^{-GL_b/\mu} + \alpha_I S^{\downarrow}$, in which, $\alpha_D(\mu)$ and α_I are the albedo of the soil for direct radiation in the direction μ and diffuse radiation, respectively.

2.2 Direct Problem Solution

To solve the ODE system, equation (2), the Euler (advanced difference) scheme of the Finite Differences Method (FDM) was used, whose computational mesh was divided into finite number of layers within the canopy as outlined in Figure 1.

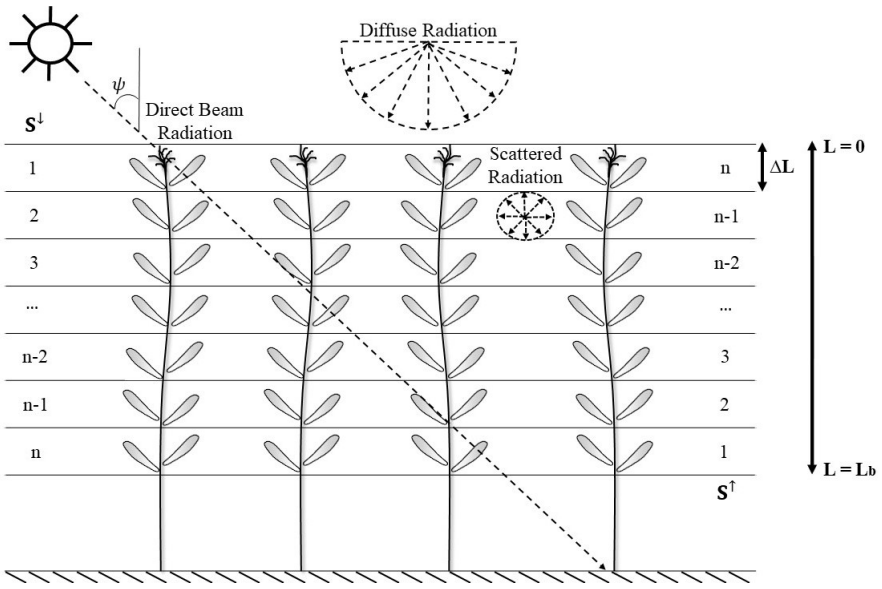

Figure 1: Spatial mesh for resolution of the ODE system.

3 The Luus-Jaakola Method

R. Luus and T. H. I. Jaakola, [6], developed a simple optimization procedure to solve nonlinear programming problems. The procedure is based on minimizing the functional given in equation (1). For that, the restrictions were defined according to the existing literature

$$MIN_{\mathbf{X}} < \mathbf{X} < MAX_{\mathbf{X}} \tag{3}$$

This is a conditional minimal problem. The Luus-Jaakola proposal is described in the algorithm below.

1. An initial random kick is generated, $\mathbf{P}^0$, within the restrictions. The amplitude of the search interval $\mathbf{r}^0 = MAX(\mathbf{P}^0) - MIN(\mathbf{P}^0)$ is denoted;

2. The residue Q_0 according to equation (1) is calculated;
3. The number of times that $\mathbf{r}^0$ will be reduced is defined;
4. A number of times that random solution candidates will be generated is also defined (parameters to be determined);
5. Random numbers are generated ($\mathbf{Y}$) between $-0,5$ and $+0,5$ for each of the parameters;
6. The random numbers from step (5) are taken and assigned to the new $\mathbf{P}$, given by

$$\mathbf{P} = \mathbf{P}^* + \mathbf{Y} \cdot \mathbf{r} \quad , \quad k = 1, \ldots, \#(\mathbf{P})$$

   where $\mathbf{P}^*$ is the best solution of the previous iteration;
7. The restrictions imposed for $\mathbf{P}$ are tested;
8. The solution of the equation (2) is calculated with the new candidates for the IP solution (parameters);
9. The new residue is calculated $Q_N(\mathbf{P})$ according to equation (1);
10. If $Q_N < Q_0$, then $Q_0 = Q_N$. Otherwise, the new values for $\mathbf{P}$ are discarded;
11. The amplitude of the search interval is reduced by a percentage pre-defined in the algorithm, called ϵ (for example 0,05), $\mathbf{r}^i = (1 - \epsilon)\mathbf{r}^{i-1} \quad 0 < \epsilon < 1$;
12. Thus, return to the step (5) until the maximum reduction of $\mathbf{r}^0$;
13. At the end of the procedure, the best $\mathbf{P}$ minimizes the functional $Q(\mathbf{P})$.

4 Results

All the tests of the external and internal iteration combinations in the inverse model, Luss-Jaakola Method, returned values for the parameters with an error less than 10^{-4} between observed and simulated data for the period used for adjustment. For each group of parameters obtained by the inverse problem, the tests in the direct model presented results for the reflected solar radiation ($S_\uparrow$) and albedo (α). Then, this was evaluated for its precision, expressed by r^2 (Figure 2), and its accuracy, represented here by the percentage RMSE (Figure 3).

Among the combinations of internal and external iterations tested, the one resulted in simulated values of more precise albedo ($r^2 = 0.7386$) was 60 and 30 external and internal iterations respectively (Figure 2(a)). However, as for the accuracy of the combination, 60/40 presented minor error (Figure 3(a)). Yet, the difference for the 60/30 combination was in the second decimal place. While the 60/40 combination had $RMSE$ equal to 6.40%, the 60/30 combination had 6.43%.

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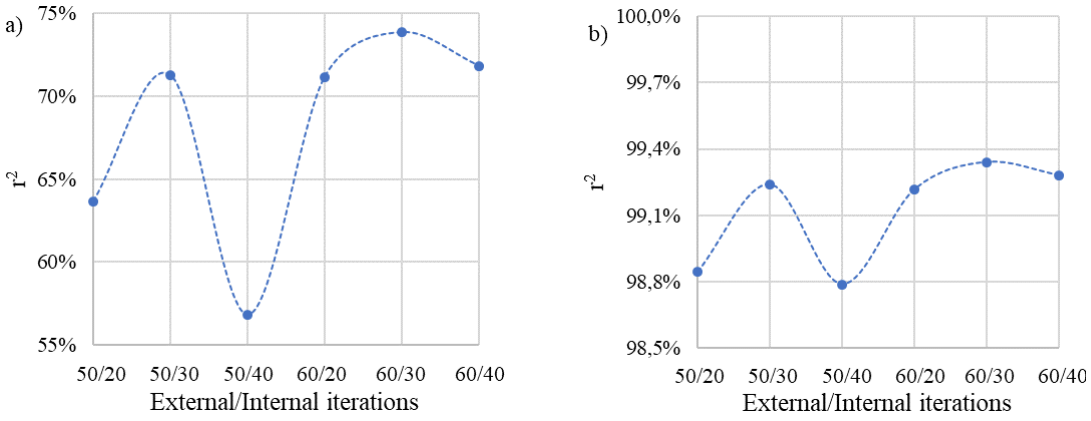
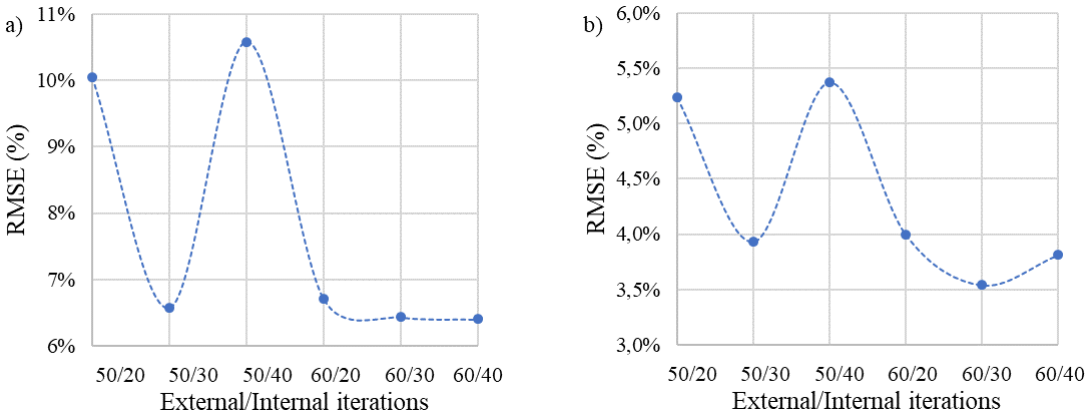


(a) albedo (b) $S^{\uparrow}$

Figure 2: Precision (r^2) of the test in the direct model with the parameters obtained.



(a) albedo (b) $S^{\uparrow}$

Figure 3: Accuracy (percentage $RMSE$) of the test in the direct model with the parameters obtained.

The precision and accuracy of $S^{\uparrow}$ was greater with the parameters obtained by the inverse problem with the combination of 60 and 30 external and internal iterations respectively (Figures 2(b) and 3(b)). Although all combinations are highly accurate, the r^2 obtained by this combination was 99.34% while the others were below 99.28%. The error, expressed by percentage $RMSE$ was small for all combinations, but the lowest was obtained with the 60/30 combination, it was equal 3.54%. Also, comparing the results of the parameters obtained with the ones found in the literature, the present values are physically coherent.

The parameters obtained by inverse problem were tested in the model in the days not used for the optimization (coefficient adjustments) and the reflected solar radiation was evaluated in relation to the established statistical indexes. Although the tests result have

an adequate representation of the observed data, in general, the $S^{\uparrow}$ estimated at the top of the canopy for this period (Figure 4) tended to underestimate the values of reflected solar radiation, especially in the early morning ($< 7\,am$). It is worth noting that Cuadra, [3], worked for sugarcane crop with the period of $8\,am$ to $5\,pm$, whereas this study used the period of $6:30\,am$ to $17\,pm$.

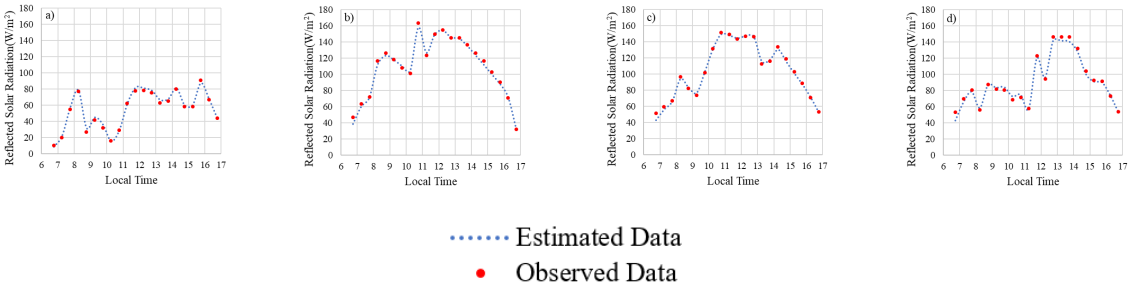


Figure 4: Evaluation of the model performance for the $S^{\uparrow}$ on the top of the canopy for each testing day: a) 02/17/2006, b) 02/18/2006, c) 02/19/2006 and d) 02/20/2006.

The albedo calculated by $S^{\uparrow}$ estimated at the top of the canopy was also evaluated. In the figure 5, the model's overestimation was observed at the lowest values of albedo on 17th and 20th of february 2006, for the days beginning at $6:30\,am$. At the beginning of the day, when the largest albedos are observed, the model underestimated. However, at these times there is less energy available. Yet, at the end of the day, when higher albedos are also observed, the model did not have problems in estimating the values.

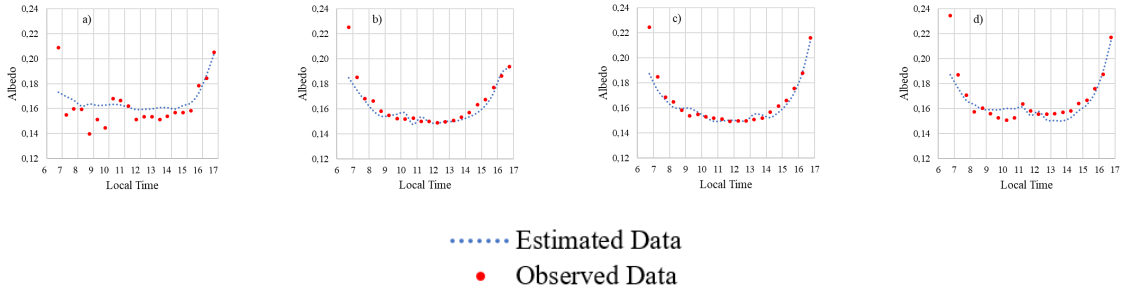


Figure 5: Evaluation of the model performance for the albedo for each testing day: a) 02/17/2006, b) 02/18/2006, c) 02/19/2006 and d) 02/20/2006.

5 Conclusions

Among the tests performed with the combinations of internal and external iterations in the Luss-Jaakola model, the result of the most accurate adjustments obtained for $S^{\uparrow}$ and α, when adjusting in relation to $S^{\uparrow}$, occurs when using 60 external and 30 internal iterations. In this case, the parameters obtained by this technique had good precision and

accuracy, of fourth order, and returned values physically coherent with the one found in the literature.

For future studies, the radiative transfer model, as well as numerical methods and the inverse problem method, should be applied to other periods of the year, that is, to use a more representative database to evaluate the generalization of the model. Moreover, It also should be applied in other vegetations with similar characteristics to its improvement and to obtain the parameters of the model to more land uses, which are complex and costly when obtained directly in the field. Although the evaluations for its generalization were not completed, this optimization provided the empirical parameters.

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