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Dynamics of a class of strongly non-linear mechanical systems

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Abstract. The aim of this work is to investigate the existence, stability and, bifurcations of periodic orbits of a strongly non-linear non-ideal mechanical problem. We have rigorously obtained the existence of periodic orbits as well as a couple of inequalities that control their stability. From these results, two new mechanical effects are obtained.

Keywords. Elliptic Functions, Perturbation Theory, Stability, Jump, Dissipation-Induced Instability

1 Introduction

In the literature on non-ideal problems, see for example [1], only weakly non-linear problems are approached. A rigorous approach to the dynamics of a weakly non-linear non-ideal problem was performed in [2]. In that paper existence, stability and bifurcations of periodic orbits, which leads to Sommerfeld Effect, were investigated. The mechanical system given in [2] is a special one, the centrifugal vibrator, but it is quite representative of all area of non-ideal mechanical problems, see [5, Ch.2]. Our goal in this work is to investigate the same questions for this mechanical system when the system is formulated as a strongly non-linear problem. This means that the unperturbed system, which is obtained when $\epsilon = 0$, is a non-linear one, see (3). It must be emphasized the existence result given in Section 3 is a dimensionless formulation of that one given in [3]. Anyway, it is necessary to repeat it here. We have rigorously obtained the existence of periodic orbits as well as a couple of inequalities which governs their stability. Such inequalities are the main mathematical result of this paper. From them, one gets two interesting mechanical effects: **a)** Jump Phenomenon, **b)** Strong dissipation-induced instability. For **a)**, in the weakly non-linear system there is the Sommerfeld Effect, see [1,2], but there is a transient regime involved in. For the present case there is no transient regime. For **b)** there is some similarity with dissipation-induced instability, see [6]. However, there are profound differences between the two cases. The details are in Section 5. Along with this research, it was necessary to do massive symbolic computations, which were performed by the CAS `Maxima`, `http://maxima.sourceforge.net/`.

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2 A Mechanical Problem

We consider a mechanical system excited by a DC motor with limited supply power, which base is supported on a spring. Besides, the DC motor rotates a small mass m, Figure 1. This mechanism is known as centrifugal vibrator.

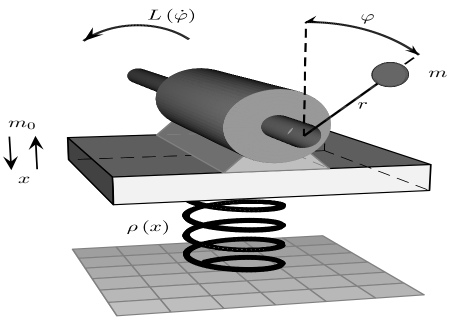

Figure 1: Centrifugal Vibrator

The mathematical model is given by the following system:

$$\begin{aligned} m_1 x'' + \beta x' + c\,x + d\,x^3 &= mr\varphi'^2 \cos\varphi + mr\varphi'' \sin\varphi, \\ I\varphi'' &= M(\varphi') + mrx'' \sin\varphi + mgr \sin\varphi \end{aligned} \tag{1}$$

where $m_1 = m_0 + m$ and m_0 denotes the mass of the DC motor. The constant c is the stiffness of the spring. And d is the elasticity coefficient that describes how much the behavior of the spring moves away from the linear case. Let P_0 be the equilibrium point of the nonlinear spring under the weight m_1. And x denotes the displacement of m_1 from P_0. The resistance of the oscillatory motion is a linear force $\beta\, x'$.

For the remainder of this paper, we consider all constants that appear in equation (1) strictly positive. We will denote by r the distance between the mass m and the axis of rotation of the DC motor. J and mr^2 are the moments of inertia of the rotating parts of the DC motor and the rotating mass m, respectively. Therefore, the total moment of inertia of the system is given by $I = J + mr^2$. Furthermore, g denotes the acceleration of gravity. The function $M(\cdot)$ is the difference between the driving torque, or characteristic, $L(\cdot)$ of the source of energy (motor) and the resistive torque $H(\cdot)$ applied to the rotor. It is assumed that $L(0) > 0$, $\frac{\partial L}{\partial \varphi'} \leqslant 0$, and $\lim_{\varphi' \to \infty} H(\varphi') = \infty$. For details see [5, pg.16-18]. Such functions $L(\cdot)$, $H(\cdot)$ are obtained from experiments. In this paper we call $M(\cdot)$ the total torque. Moreover, take for granted that $\varphi' > 0$.

We can rewrite the equations of motion (1) as a dimensionless first order system. Take

$$\begin{aligned} &x_1(t) = x(t), x_2(t) = x'(t), x_3(t) = \varphi(t), x_4(t) = \varphi'(t), \\ &s = \sqrt{\tfrac{d}{m_1}}\, r\, t, y_1(s) = \tfrac{1}{r}\, x_1\left(\tfrac{\sqrt{m_1}\, s}{\sqrt{d}\, r}\right), y_2(s) = \tfrac{\sqrt{m_1}}{\sqrt{d}\, r^2}\, x_2\left(\tfrac{\sqrt{m_1}\, s}{\sqrt{d}\, r}\right), \\ &y_3(s) = x_3\left(\tfrac{\sqrt{m_1}\, s}{\sqrt{d}\, r}\right), y_4(s) = \tfrac{\sqrt{m_1}}{\sqrt{d}\, r}\, x_4\left(\tfrac{\sqrt{m_1}\, s}{\sqrt{d}\, r}\right), M_1(z) = \tfrac{m_1}{d\, r^2\, J}\, M\left(\sqrt{\tfrac{d}{m_1}}\, r\, z\right). \\ &a_1 = \tfrac{c}{d\, r^2}, a_2 = \tfrac{\beta}{\sqrt{d}\, \sqrt{m_1}\, r}, a_3 = \tfrac{m}{m_1}, a_4 = \tfrac{J}{I}, a_5 = \tfrac{m\, r^2}{I}, a_6 = \tfrac{g\, m\, m_1}{d\, r\, I}. \end{aligned} \tag{2}$$

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Observe that $a_4 + a_5 = 1$. Moreover, let us introduce a small parameter ϵ in the dimensionless parameters and total torque. From the heuristic point of view if c, β, m are small then the dimensionless parameters $a_i, i \neq 4$ in (2) are small too. So, let us substitute a_i, $i \neq 4$ by ϵa_i, a_4 by $1 - \epsilon a_5$ and M_1 by ϵM_1. Thus, performing such substitutions into (1) and using (2), one gets

$$\begin{aligned}
y_1'(s) &= y_2(s), \\
y_2'(s) &= -y_1(s)^3 + \epsilon\left(a_3\, y_4(s)^2 \cos(y_3(s)) - a_2\, y_2(s) - a_1\, y_1(s)\right) + O\left(\epsilon^2\right), \\
y_3'(s) &= y_4(s), \\
y_4'(s) &= \epsilon\left(M_1(y_4(s)) + \left(a_6 - a_5\, y_1(s)^3\right)\sin(y_3(s))\right) + O\left(\epsilon^2\right).
\end{aligned} \tag{3}$$

Let us consider the following change of variables for (3),

$$y_1(s) = C(s)\ cn\left(D(s), \tfrac{1}{\sqrt{2}}\right), \quad y_2(s) = C(s)^2\ cn'\left(D(s), \tfrac{1}{\sqrt{2}}\right) \tag{4}$$

where $cn\left(\cdot, 1/\sqrt{2}\right)$ denotes the jacobian cosine with modulus $1/\sqrt{2}$, and k_0 indicates its period. For details see [9]. Hence the system (3) can be rewritten in the variables C, D, y_3, y_4. Since this system is autonomous, by assuming $y_4(s) \neq 0$, one can perform the usual reduction of order in this last system. Such reduced system is a third order one and is written in the variables $\overline{C}, \overline{D}, \overline{y}_4$ and the time variable is denoted by u. Now, let us use the following change of variables $\overline{D}(u) = \overline{D}_1(u) - \frac{k_0\, u}{2\pi}$. So, one finally gets the following time-dependent system

$$\begin{aligned}
\overline{C}'(u) &= \tfrac{\epsilon}{2\,\overline{y}_4(u)\,\overline{C}(u)}\Bigg(a_2\,\overline{C}(u)^2\left(cn^4\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right) - 1\right) \\
&\quad -2\,a_1\,\overline{C}(u)\ cn'\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right)\ cn\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right) \\
&\quad +2\,a_3\, y_4(u)^2 \cos(u)\ cn'\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right)\Bigg) + O\left(\epsilon^2\right) \\
\overline{D}_1'(u) &= \tfrac{2\pi\,\overline{C}(u) + k_0\,\overline{y}_4(u)}{2\pi\,\overline{y}_4(u)} + \tfrac{\epsilon}{\overline{y}_4(u)\,\overline{C}(u)^2}\Bigg(a_1\,\overline{C}(u)\ cn^2\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right) \\
&\quad +a_2\,\overline{C}(u)^2\ cn'\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right)\ cn\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right) \\
&\quad -a_3\,\overline{y}_4(u)^2 \cos(u)\ cn\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right)\Bigg) + O\left(\epsilon^2\right) \\
\overline{y}_4'(u) &= \tfrac{\epsilon}{\overline{y}_4(u)}\left(a_5\,\overline{C}(u)^3 \sin(u)\ cn^3\left(\tfrac{2\pi\,\overline{D}_1(u) - k_0\, u}{2\pi}, \tfrac{1}{\sqrt{2}}\right) - M_1\left(\overline{y}_4(u)\right) - a_6 \sin(u)\right) \\
&\quad + O\left(\epsilon^2\right)
\end{aligned} \tag{5}$$

Note that (5) is 2π-periodic in the variable u.

3 Existence of Periodic Orbits

Assume

$$\left| \frac{k_0^4\, a_2 \cosh\left(\frac{\pi}{2}\right)}{3\, 2^{\frac{9}{2}}\, \pi^4\, a_3} \right| < 1 \tag{6}$$

and that

$$M_1 \left(-\frac{2\,\pi\,\overline{a}}{k_0} \right) = -\frac{k_0\, a_2\, a_5\, \overline{a}^3}{6\,\pi\, a_3} \tag{7}$$

has negative real roots. The signal of $\overline{a}$ is due to the condition $\varphi' > 0$. In view of (6) there is $\overline{b}$ such that

$$\sin\left(\frac{2\,\pi\,\overline{b}}{k_0}\right) = -\frac{k_0^4\, a_2 \cosh\left(\frac{\pi}{2}\right)}{3\, 2^{\frac{9}{2}}\, \pi^4\, a_3}. \tag{8}$$

By using the Poincaré Method, see [3], one gets that if

$$M_1' \left(-\frac{2\,\pi\,\overline{a}}{k_0} \right) \neq \frac{k_0^2\, a_2\, a_5\, \overline{a}^2}{4\,\pi^2\, a_3} \tag{9}$$

holds, then there are $\epsilon_0 > 0$ and C^∞ functions $a, b, c : (-\epsilon_0, \epsilon_0) \to \mathbb{R}$ with $a(0) = \overline{a}, b(0) = \overline{b}, c(0) = \overline{c}$ where $\overline{c} = \overline{c}\left(\overline{a}, \overline{b}\right)$ is a complicated function of $\overline{a}$ and $\overline{b}$, such that for all $\epsilon \in (-\epsilon_0, \epsilon_0)$ the solution of (5) with initial conditions $\overline{C}(0) = a(\epsilon)$, $\overline{D}_1(0) = b(\epsilon)$, $\overline{y}_4(0) = -\frac{2\pi\overline{a}}{k_0} + \epsilon\, c(\epsilon)$ is a 2π-periodic orbit.

4 Stability of the Periodic Solution

From now on let us use the variable s instead of u for the time. Let us denote by $z_0(\cdot,\epsilon)$ the 2π-periodic solution of (5) which initial conditions are given in the Section 3. By using Regular Perturbation Theory, one gets the expansion of this solution to any order of ϵ. The computations are long but straightforward. The linearization of (5) at $z_0(\cdot,\epsilon)$ leads to the following system $\mathbf{U}'(s) = \mathbf{A}(s,\epsilon)\, \mathbf{U}(s)$ where $\mathbf{U}(s) = (U_1(s), U_2(s), U_3(s))$ and $\mathbf{A} : \mathbb{R} \times (-\epsilon_0, \epsilon_0) \to L\left(\mathbb{R}^3, \mathbb{R}^3\right)$ is C^∞ mapping and is $2\,\pi$-periodic in the s variable. Of course, one can use Regular Perturbation Theory here in order to compute an expansion of the monodromy matrix in terms of ϵ. Thus let $\mathbf{N}(s,\epsilon)$ be the the principal matrix solution of the following matrix diferential equation $\mathbf{N}'(s,\epsilon) = \mathbf{A}(s,\epsilon)\, \mathbf{N}(s,\epsilon)$ The monodromy matrix is the following one $\mathbf{M}(\epsilon) = \mathbf{N}(2\,\pi,\epsilon)$. For oomputation of the eigenvalues of $\mathbf{M}$, consider the polynomial $P(\epsilon, X) = -\det(\mathbf{M}(\epsilon) - X\,\mathbf{I})$. This polynomial has huge coefficients, but after an extense computation, one gets

$$\frac{P\left(\epsilon^2, 1+\epsilon\, Y\right)}{\epsilon^3} = Y^3 + c_2(\epsilon)\, Y^2 + c_1(\epsilon)\, Y + c_0(\epsilon)\,. \tag{10}$$

Let us denote by $P_1(\epsilon, Y)$ the right hand-side of (10). One has that $P_1(\epsilon, Y) = P_1(0, Y) + \epsilon\, Q(\epsilon, Y)$, where Q is second degree polynomial in the variable Y. By assuming that $\cos\left(\frac{2\,\pi\,\overline{b}}{k_0}\right) \neq 0$ the roots of $P_1(0, Y)$ are $0, \pm r_1$ where $r_1 = w_1 \sqrt{-w_2} \big/ w_3$, $w_1 = 2^{\frac{7}{4}}\, \pi^{\frac{3}{2}}$,

$w_2 = \left(k_0^2\, a_5\, \overline{a}^2 + 4\,\pi^2\, a_3\right)\, \cos\left(\frac{2\,\pi\,\overline{b}}{k_0}\right) \diagup \overline{a}$, $w_3 = k_0^{\frac{3}{2}}\, \sqrt{\cosh\left(\frac{\pi}{2}\right)}$. Here the Implicit Function Theorem can be applied to this polynomial and one gets expansions of the roots to any order in ϵ. For the root 0 in $P_1\,(0, Y) = 0$, the corresponding root of $P_1\,(\epsilon, Y)$ is denoted by $Y\,(\epsilon)$ and $Y\,(0) = 0$. It follows from (10) that $X_{1\,real}\,(\epsilon) = 1 + \sqrt{\epsilon}\, Y\,(\sqrt{\epsilon})$ is the root of $P\,(\epsilon, X) = 0$ such that $X_{1\,real}\,(0) = 1$. From the expansion of $Y\,(\epsilon)$ one obtains

$$X_{1\,real}\,(\epsilon) = 1 - \epsilon\, \frac{4\,\pi^2\, k_0\, a_3\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - k_0^3\, a_2\, a_5\, \overline{a}^2}{k_0^2\, a_5\, \overline{a}^3 + 4\,\pi^2\, a_3\, \overline{a}} + O\left(\epsilon^{\frac{3}{2}}\right). \tag{11}$$

For the roots $\pm r_1$ one has to take into account two cases:

$$\frac{\cos\left(\frac{2\,\pi\,\overline{b}}{k_0}\right)}{\overline{a}} < 0, \tag{12}$$

$$\frac{\cos\left(\frac{2\,\pi\,\overline{b}}{k_0}\right)}{\overline{a}} > 0. \tag{13}$$

If (12) holds and using the same way for the obtaining of (11), one gets two real roots given by $X_{2\,real}\,(\epsilon) = 1 - r_1\,\sqrt{\epsilon} + O\,(\epsilon)$, $X_{3\,real}\,(\epsilon) = 1 + r_1\,\sqrt{\epsilon} + O\,(\epsilon)$. If (13) holds, one has two conjugated complex roots denoted by $X_{2\,complex}\,(\epsilon)$, $X_{3\,complex}\,(\epsilon)$ respectively such that

$$|X_{2\,complex}\,(\epsilon)\,|^2 = 1 - \epsilon\, \left(\frac{k_0\,\left(k_0^2\, a_5\, \overline{a}^2\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - 4\,\pi^2\, a_2\, a_3\right)}{\overline{a}\,\left(k_0^2\, a_5\, \overline{a}^2 + 4\,\pi^2\, a_3\right)}\right) + O\left(\epsilon^{\frac{3}{2}}\right). \tag{14}$$

Then, if (12) holds and since $X_{3\,real}\,(\epsilon) > 1$, it follows from [4, Th.3.1,pg.157] that the periodic orbit $z_0\,(\cdot,\epsilon)$ is unstable. If (13) holds one gets from (11) and (14) that if

$$\frac{4\,\pi^2\, a_3\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - k_0^2\, a_2\, a_5\, \overline{a}^2}{\overline{a}} > 0 \quad \text{and} \quad \frac{k_0^2\, a_5\, \overline{a}^2\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - 4\,\pi^2\, a_2\, a_3}{\overline{a}} > 0 \tag{15}$$

then $|X_{1\,real}\,(\epsilon)\,| < 1$, $|X_{2\,complex}\,(\epsilon)\,| = |X_{3\,complex}\,(\epsilon)\,| < 1$ so the periodic orbit $z_0\,(\cdot,\epsilon)$ is asymptotically stable. The proof of this result is exactly the same as for the unstable case. By using the same argument, one has if

$$\frac{4\,\pi^2\, a_3\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - k_0^2\, a_2\, a_5\, \overline{a}^2}{\overline{a}} < 0 \quad \text{or} \quad \frac{k_0^2\, a_5\, \overline{a}^2\, M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right) - 4\,\pi^2\, a_2\, a_3}{\overline{a}} < 0 \tag{16}$$

one gets $|X_{1\,real}\,(\epsilon)\,| > 1$, or $|X_{2\,complex}\,(\epsilon)\,| = |X_{3\,complex}\,(\epsilon)\,| > 1$ then $z_0\,(\cdot,\epsilon)$ is an unstable periodic orbit. Summing up our main result on stability is the following one:

Theorem 4.1. *Let $z_0\,(\cdot,\epsilon)$ be the 2π-periodic orbit of* (5) *obtained in Section 3.*

a) If (12) *holds the periodic orbit $z_0\,(\cdot,\epsilon)$ is unstable.*

b) If (13), (15) *hold the periodic orbit $z_0\,(\cdot,\epsilon)$ is asymptotically stable.*

c) If (13), (16) *hold the periodic orbit $z_0\,(\cdot,\epsilon)$ is unstable.*

5 Some Mechanical Effects

In this section, two physical effects are obtained from the use of Theorem 4.1. These are the following ones: a) Jump Phenomenon, b) Strong dissipation-induced instability. Due to the lack of space, only a sketch of these results is given.

a) The Jump Phenomenon

In (5) let us rewrite a_2 in terms of $\overline{b}$ by using (8). Of course, it is assumed that (6) holds. Note that the parameter a_2 is determined by the dissipation, see (2). Observe the initial conditions of the periodic orbit obtained for (5) are given at the end of Section 3 and are the following ones $\overline{C}\,(0) = a\,(\epsilon)$, $\overline{D}_1\,(0) = b\,(\epsilon)$, $\overline{y}_4\,(0) = -\frac{2\pi\overline{a}}{k_0} + \epsilon c\,(\epsilon)$, where $b\,(0) = \overline{b}$. So, the zero order term of the second initial of the periodic orbit, obtained in Section 3, is a parameter of the system (5). Since $\sin\left(\frac{2\pi\,\overline{b}}{k_0}\right) < 0$ one obtains $\overline{b} \in \left(\frac{k_0}{2}, \frac{3\,k_0}{4}\right) \cup \left(\frac{3\,k_0}{4}, k_0\right)$. By using the change of variables (4), the dimensionless parameters given at (2), and Theorem 4.1 one concludes that if $\overline{b} \in \left(\frac{k_0}{2}, \frac{3\,k_0}{4}\right)$ the periodic orbit is stable and one has $x\,(0) > 0,\, x'\,(0) > 0$. And if $\overline{b} \in \left(\frac{3\,k_0}{4}, k_0\right)$ the periodic orbit is unstable and one has $x\,(0) < 0,\, x'\,(0) > 0$. So, when $\overline{b} \in \left(\frac{k_0}{2}, \frac{3\,k_0}{4}\right)$ and $M_1'\left(-\frac{2\,\pi\,\overline{a}}{k_0}\right)$ is adequately small, the periodic orbit is stable and the centrifugal vibrator is placed above of the equilibrium of the nonlinear spring. If $\overline{b} \in \left(\frac{3\,k_0}{4}, k_0\right)$ there is instability of the periodic orbit and the centrifugal vibrator suffers a jump and is replaced in a position below of the equilibrium of the nonlinear spring.

Consider the linearization of (5) at the periodic orbit $z_0\,(\cdot,\epsilon)$. By taking into account the parameter $\overline{b}$ in the linearized equation one has $\mathbf{U}'\,(s) = \mathbf{A}\left(s,\,\epsilon,\overline{b}\right)\,\mathbf{U}\,(s)$. In view of [8, Theorem 12,pg.146] the last system is equivalent to $\mathbf{V}'\,(s) = \mathbf{B}_1\left(\epsilon,\overline{b}\right)\,\mathbf{V}\,(s)$ where $\mathbf{B}_1\left(\epsilon,\overline{b}\right)$ is a real matrix. For $\overline{b} \in \left(\frac{k_0}{2}, \frac{3\,k_0}{4}\right)$, in view of Theorem 4.1, all characteristic multipliers have norm lesser than 1. So, the correspondent characteristic exponents have negative real part. For $\overline{b} \in \left(\frac{3\,k_0}{4}, k_0\right)$ and proceeding analogously as in the previous argument, there is a real characteristic multiplier which norm is greater than 1. Hence the correspondent characteristic exponent has positive real part. Since $\mathbf{B}_1\left(\epsilon,\overline{b}\right)$ is a differentiable function of $\overline{b}$, then $\mathbf{B}_1$ has the eigenvalue zero at $\overline{b} = \frac{3\,k_0}{4}$. This bifurcation is called a fold one, see [7, pg.80]. As an example, take the following total torque $M_1\,(x) = A - B\,x$ where $A > 0$ and $B > 0$. From above argument one concludes the existence of a fold bifurcation for this total torque.

b) A case of Strong dissipation-induced instability

Consider the following total torque $M_1\,(x) = -x^3 + x^2 - C\,x + C$, where C is a positive constant, $L\,(x) = -x^3 + x^2 + C$, $H\,(x) = C\,x$. Assume that $\overline{b} \in \left(\frac{k_0}{2}, \frac{3\,k_0}{4}\right)$. It can be proved that if a_2 is near 0 then the orbit is stable, but if $a_3 << 1$, $1 - a_5 << 1$, and a_2 is near of its maximum value, given by $\frac{3\,2^{\frac{9}{2}}\,\pi^4\,a_3}{k_0^4\cosh\left(\frac{\pi}{2}\right)}$, then this periodic orbit is unstable. This means, for this case, that an adequate increase of dissipation leads to instability. We call it strong dissipation-induced instability. This counter-intuitive phenomenon is similar to one known in the literature as dissipation-induced instability, see [6]. But there are some striking differences. In [6] the linearization is performed around equilibrium points. Here we deal with periodic orbits. The systems involved in [6] are, in its beginning, conservative ones. Here the system (5) is a dissipative one. In [6], an infinitesimal increase of the dissipation of leads from a stable equilibrium point to an unstable one. In our case the periodic orbit becomes unstable only after a finite amount of dissipation.

There is no common framework for both kinds of phenomena yet.

6 Conclusions

In this paper, the dynamics of a strongly non-linear non-ideal system, the centrifugal vibrator, is investigated. We have gotten results on existence, stability, and bifurcations of periodic orbits. From that two new mechanical phenomena are shown. The Sommerfeld Effect, for this case, was not obtained as in the weakly non-linear case due to the absence of a transient regime.

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