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Analysis of Forced Vibrations in two Degrees of Freedom Structures

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1 Introduction

Structures in Civil Engineering are constantly subject to the most diverse actions of external forces like loads of people and vehicles, actions of the winds, earthquakes, explosions and others [1, 2]. The appropriate understanding of the response to external excitations is a fundamental way to optimize the civil construction processes as well as to improve the safety factors associated with each component of the structure. In recent years, the advance of the computational processing capacity has propelled the implementation of several studies aimed to understand the dynamic response of several mechanical complex systems to the more diverses [3–5].

In this work we analyze the dynamic behavior of a two-story structure, as showed in Figure 1(a), under the influence of mechanical vibrations caused by the action of external forces arising from the several vertical loads to which civil engineering structures are normally subject.

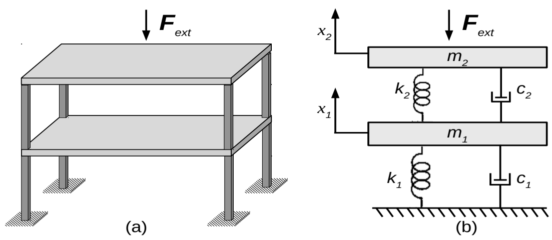

Figura 1: Structure with two floors subject to external force action (a). Simplified mathematical model in (b).

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2 Dynamical Model

The precise dynamic description of complex structural systems is closely related to the number of parameters required to obtain the necessary information. In this perspective, the number of degrees of freedom corresponds to the number of independent variables necessary for the complete and unambiguous description of the state of the system at each instant of time. In Figure 1(b), a two degree of freedom a mathematical model simplified is shown. The variables x_1 e x_2 describe the state of the system at each time. The differential equation governing the dynamics of this system is given by the second Newton's law as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{1}$$

In this work the elastic (k) and damping (c) coefficients are related to the mechanical characteristics of the materials composing the pillars, such as the Young's modulus E and the Poison Coefficient ν. The natural frequencies of this system provide important information that can be used to determine which external force cases promote, for example, resonance and structure collapse. They can be written as:

$$\omega_{1,2}^2 = \frac{k_2 m_1 + (k_1 + k_2) m_2}{2 m_1 m_2} \mp \left[\left(\frac{k_2 m_1 + (k_1 + k_2) m_2}{4 m_1 m_2} \right)^2 - \frac{k_1 k_2}{m1m2} \right]^{1/2} \tag{2}$$

3 Final Remarks

Considering a scenario close to reality, the system was solved numerically considering the physical properties of different pillars composed of reinforced concrete, wood and metal. The obtained results show which conditions promote the different behaviors like harmonic motion, simple and dampped, beats and resonance.

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