Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 7, n. 1, 2020.

Trabalho apresentado no XXXIX CNMAC, Uberlândia - MG, 2019.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

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Lagrangian formulation applied in a non-linear variant of the Atwood Machine

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1 Abstract

The Atwood machine is a common tool for mechanical systems validation, therefore the increased complexity is an alternative for testing modelling techniques and numerical methods. Thus, in this paper, there is presented a variant formulated through the Lagrangean paradigm and solved using Runge-Kutta fourth order numerical method. The system numerical characteristics were conveniently chosen to obtain the desired of results.

2 Introduction

George Atwood proposed in 1784 the mechanism known as Atwood machine, consisting of two distinct masses (m_1 and m_2), connected through an inextensible cable supported by an ideal pulley [1]. We developed a deviated system, composed of a third mass (m_3) connected to the second mass over a non-linear spring, modelled by the Duffing equation [2], a viscous damper and inertia in the pulley now has inertia. The system is presented in Figure 1.

With the Lagrangian formalism it is possible to obtain a system of differential equations. With the Lagrangian function, that is defined by the difference of the potential energy from the kinetic energy, that equating the non-conservative energy.

Thus we have the Lagrangian equation, where ϵ is defined as $x_3 - x_2 - l$, being l and l_o the unstressed spring and cable length respectively, g the applied gravity, m_r the pulley mass the and α, β and k are the relative therms of the Duffing Equation [2]:

$$L = \frac{m_1 + m_2 + 0.5m_r}{2}\dot{x_2}^2 + \frac{m_3}{2}\dot{x_3}^2 + (m_2 - m_1)gx_2 + m_1 g l_o + mgx_3 - \frac{k}{2}\epsilon^2 - \frac{\beta}{3}\epsilon^3 - \frac{\alpha}{4}\epsilon^4 \quad (1)$$

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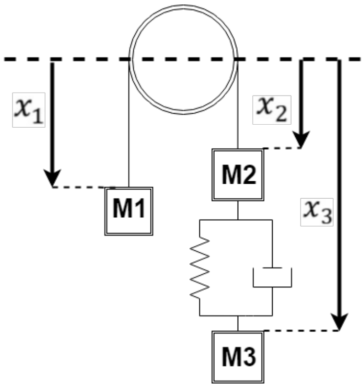
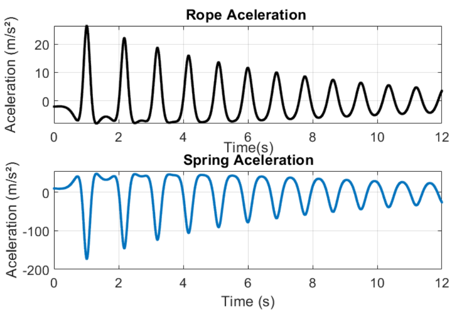


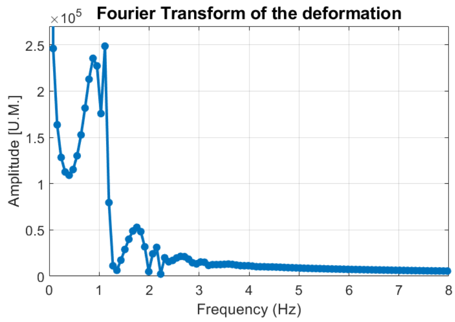
Figure 1: Atwood Machine Variant

3 Development

The accelerations were solved using the Runge-Kutta fourth order method [?]. Their results and the Fourier Transform using the fallowing parameters: $l_o = 5$, $l = 70$, $X_2 = 40$ $m_1 = 150$, $m_2 = 95$, $m_3 = 40$, $m_r = 20$, $k = 50$, $\alpha = 50$, $\beta = -300$, $c = 10$ and $g = 9.80665$, all in SI units, were represented in 2.



(a) Accelerations



(b) Deformation Fourier Transform

Figure 2: System Characteristics

On the first graph the amplitude of vibration decays as result of the energy loss through the damper. The second graph displays oscillation in multiple frequencies, presenting its non-linearity. In a linear system the oscillation occurs in its natural frequency, in a Duffing equation model the free vibration will occur in multiple frequencies [2].

References

[1] ATWOOD G., **A treatise on the rectilinear motion and rotation of bodies; with a description of original experiments.**, Cambridge, 1784.

[2] KOVACIC I. and BRENNAN M. J., **The Duffing Equation: Nonlinear Oscillators and their Behaviour** , Wiley, 2011.