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A Delay Differential Equations model for *Wolbachia* infection in *Aedes aegypti* populations [1]

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1 Introduction

Currently, *Wolbachia* is one of the most studied reproductive parasite of arthropod species and appears to be a promising candidate for biocontrol of some mosquito carried diseases, especially by *Aedes aegypti*. A mathematical model inspired by Nicholson-type delay equations is proposed to investigate the relation between temperature variation (implicit into maturation time) and population dynamics of both transinfected by a Wolbachia strain and uninfected *Aedes aegypti* mosquitoes. We analyze the existence and local stability of equilibria with and without delay.

2 Model

Let w and n be, respectively, the number of *Wolbachia* infected and uninfected individuals. Our model consists in

$$\begin{aligned} \dot{w}(t) &= P_w e^{-\mu\tau} w(t-\tau) e^{-(w(t-\tau)+n(t-\tau))^\eta} - \delta_w w(t), \\ \dot{n}(t) &= \left(P_n n(t-\tau) + P_{nw} w(t-\tau)\right) e^{-\mu\tau} e^{-(w(t-\tau)+n(t-\tau))^\eta} - \delta_n n(t), \end{aligned} \tag{1}$$

being $P_w, P_{nw}, P_n, \bar{\mu}, \mu, \tau, \eta, \delta_w, \delta_n > 0$ and

$$(w(s), n(s)) = (\varphi_w(s), \varphi_n(s)), \quad s \in [-\tau, 0], \tag{2}$$

where $\varphi_w, \varphi_n : [-\tau, 0] \to [0, \infty)$ are continuous.

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The immature development time (duration of aquatic phase of mosquito) τ and the death rate of the immature phase μ are the same for both infected and uninfected populations since the *Wolbachia* infection does not affect the immature stage [4]. P_w denotes the birth rate for infected newborns generated by infected adults, P_{nw} the birth rate for uninfected newborns generated by infected adults and P_n the birth rate for uninfected newborns generated by uninfected adults. Fertility depends on the number of individuals at time $t-\tau$, namely $w(t-\tau)$ and $n(t-\tau)$. Also, $e^{-\mu\tau}$ takes into account an individual survival up to age τ [2, 3]. Interspecific competition is modelled by $e^{-(w(t-\tau)+n(t-\tau))^{\eta}}$, where η measures how rapidly carrying capacity is achieved [1]. Last, δ_w and δ_n are the mortality rates of adult infected and uninfected mosquitoes, respectively.

3 Previous results

The model presents the positive, *Wolbachia*-free and trivial steady states for the system: $\mathbf{S_2} = \left(\left[\ln\left(\frac{P_w}{\delta_w}\right) - \mu\tau\right]^{\frac{1}{\eta}} \frac{\delta_n P_w - \delta_w P_n}{\delta_n P_w + \delta_w (P_{nw} - P_n)}, \left[\ln\left(\frac{P_w}{\delta_w}\right) - \mu\tau\right]^{\frac{1}{\eta}} \frac{\delta_w P_{nw}}{\delta_n P_w + \delta_w (P_{nw} - P_n)}\right)$, $\mathbf{S_1} = \left(0, \left[\ln\left(\frac{P_n}{\delta_n}\right) - \mu\tau\right]^{\frac{1}{\eta}}\right)$ and $\mathbf{S_0} = (0, 0)$.

By determining the thresholds τ_0, τ_1 and τ_2, we prove that: i) $\mathbf{S_0}$ always exists; ii) for $\tau < \tau_1$, $\mathbf{S_1}$ is possible; iii) for $\tau < \tau_2$, $\mathbf{S_2}$ is possible since $\tau_1 < \tau_2$; iv) if $\mathbf{S_0}$, $\mathbf{S_1}$ and $\mathbf{S_2}$ or just $\mathbf{S_0}$ and $\mathbf{S_2}$ coexist, $\mathbf{S_2}$ is locally stable, vi) if $\mathbf{S_0}$ and $\mathbf{S_1}$ coexist, then $\mathbf{S_1}$ is locally stable. Moreover, based on well established fact that the development time varies conversely with temperature, we show that for low temperatures ($\tau > \tau_0$) the only possibility is the extinction of both mosquito populations (global stability of $\mathbf{S_0}$).

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References

[1] C. Dye, Models for the Population Dynamics of the Yellow Fever Mosquito, Aedes aegypti. *Journal of Animal Ecology*, 53: 247-268, 1984.

[2] M. Iannelli, A. Pugliese, *An Introduction to Mathematical Population Dynamics*, Spring, 2014.

[3] M. J. Keeling, F. M. Jiggins, J. M. Read, The invasion and coexistence of competing Wolbachia strains, *Heredity*, 91: 382-8, 2003.

[4] M. Ndii, m., R. Hickson, G. Mercer, Modelling the introduction of Wolbachia into Aedes aegypti mosquitoes to reduce dengue transmission, *The ANZIAM Journal*, 53(3): 213-227, 2012.