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Fractional order log barrier interior point method

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1 Introduction

Interior point methods are based on a trajectory construction in a feasible region, from an initial given point, being widely used for the solution of convex constrained optimization problems such as economic dispatch [3], optimum power flow [5], loss minimization, fitting curves [2] and many others. One of the limitations of Newton's method for minimizing convex functions is that we cannot deal with inequality constraints, however, the log barrier method is a way to address this issue.

The fractional calculus [4] has been employed for solving optimization problems [1]. However, such investigations began only recently. In particular, the Caputo left-sided fractional derivative of order $\alpha > 0$, is defined by

$$\left({}^{\mathrm{C}}\mathrm{D}^{\alpha}_{a+}f\right)(x) := \frac{1}{\Gamma(n-\alpha)}\left(\int_a^x \frac{f^{(n)}(\tau)}{(x-\tau)^{1-n+\alpha}}\,\mathrm{d}\tau\right), \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function and[3] $n = [\alpha] + 1$.

2 Log barrier interior point method

Consider the following problem

$$\begin{array}{rl} \min & f(x) \\ \text{subject to} & g_i(x) \geq 0 \quad (i = 1, 2, \ldots, m), \\ & \mathrm{A}x = b, \end{array} \tag{2}$$

where $f, g_i : \mathbb{R}^n \to \mathbb{R}$ are all convex and $f, g_i \in \mathrm{C}^2(\mathbb{R}^n)$. The log barrier function is defined as $\phi(x) = -\mu \sum_{i=1}^{m} \ln(g_i(x))$, where μ, with $\mu \to 0$, is a barrier parameter. The domain of the $\phi(x)$ is the set of strictly feasible points $\{x \in \mathbb{R}^n / \, g_i(x) > 0, \; i = 1, 2, \ldots, m\}$.

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[3] $[\mu]$ indicates the integer part of μ.

The idea of the log barrier is

$$\begin{array}{rl} \min & f(x) - \mu \sum_{i=1}^{m} \ln(g_i(x)) \\ \text{subject to} & \mathrm{A}x = b. \end{array} \tag{3}$$

We intend to formulate the fractional order log barrier interior point method to solve the L_p-norm minimization problem, when $1 < p < 2$ (super-Gaussian) and $2 < p < \infty$ (sub-Gaussian), given by

$$\min_{x \in \mathbb{R}^n} \quad \|\mathrm{A}x - b\|_p^p, \tag{4}$$

where the matrix $\mathrm{A} \in \mathbb{R}^{m \times n}$ has full rank, $b \in \mathbb{R}^m$ and $m > n$. This problem can be rewritten in the form

$$\begin{array}{rl} \min & \phi(u,v) = \sum_{i=1}^{m} (u_i + v_i)^p \\ \text{subject to} & \mathrm{A}x + u - v - b = 0, \\ & (u,v) \geq 0. \end{array} \tag{5}$$

We can rewrite our problem in equation (5) as

$$\begin{array}{rl} \min & \sum_{i=1}^{m} (u_i + v_i)^p - \mu \sum_{i=1}^{m} \ln(u_i) - \mu \sum_{i=1}^{m} \ln(v_i) \\ \text{subject to} & \mathrm{A}x + u - v - b = 0. \end{array} \tag{6}$$

3 Conclusions

We aim to implement an algorithm to solve the problem in equation (6) by using Caputo fractional derivative, defined in equation (1), in the necessary and sufficient conditions given by the KKT conditions in log barrier interior point method, compare it with the classical method, and apply the new method for fitting curves.

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