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Less Conservative LMI-Based Conditions to Analyze the Solution of Switched TS Fuzzy Systems

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Abstract. In this paper, a less conservative sufficient condition that ensure the ultimate boundedness of the solutions of switched TS fuzzy systems is proposed. The main result is given in terms of linear matrix inequalities (LMIs), which were formulated by calculating the derivative of the auxiliary function V along the solution of the system formed by a convex combination of the switching subsystems. The LMI conditions were relaxed using the S-procedure. A numerical example illustrates the efficiency of the proposed result.

Keywords. Switched TS fuzzy system, LMIs, Ultimate Boundedness, S-procedure.

1 Introduction

Switched system has been used to model many applications in several areas [3, 6, 7, 10]. Due to the large number of applications, the theory for this class of system has attracted the attention of many researchers. In consequence, many approaches for stability analysis and controller synthesis were developed [1, 8, 9, 12]. However, the results presented in these papers depend on the existence of a Lyapunov function or a Lyapunov-like function, which it is not an easy task. In order to overcome this problem, in [11] sufficient LMI conditions for the ultimate boundedness of the solutions of switched TS fuzzy systems were presented. The main feature of the result is that the derivative of the function can assume positive values in bounded sets. In this paper, we provide less conservative conditions than presented in [11], by exploring the S-procedure. The S-procedure is a mathematical result which provides information about a specific quadratic inequality by analyzing another quadratic inequality. More details on this topic can be found in [2, 4, 5, 7]. Henceforth, $\bar{\Omega}$ denotes the closure of set Ω, the notation $\mathbf{P} \succ \mathbf{0}$ ($\mathbf{P} \succeq \mathbf{0}$) indicates that $\mathbf{P}$ is a real symmetric and positive definite (semi-definite) matrix and the notation $\mathbf{P} \prec \mathbf{0}$ ($\mathbf{P} \preceq \mathbf{0}$)

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indicates that $\mathbf{P}$ is a symmetric and negative definite (semi-definite) matrix, the symbol "$\star$" within a matrix represents the symmetric terms of the matrix and e_i denotes a vector with one at entry i and zeros elsewhere, that is, $e_i = [0 \ \cdots \ 0 \underbrace{1}_{i-th} 0 \ \cdots \ 0]' \in \mathbb{R}^n$ where $'$ denotes the transposed vector, ∂Z denotes the boundary of set Z, and finally, inf and sup denote the infimum and supremum of a subset, respectively.

2 Preliminaries

Let us consider the following switched TS fuzzy system

$$\dot{\mathbf{x}}(t) = \sum_{k=1}^{r} h_{\sigma(x(t))k}(x(t))\mathbf{A}_{\sigma(x(t))k}x(t) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state, $\sigma(x(t)) : \mathbb{R}^n \to \mathcal{P}$ is a piecewise constant function of the state called switching signal, with $\mathcal{P} = \{1, 2, \cdots, N\}$ and N the number of subsystems; $\mathbf{A}_{\sigma(x(t))k} \in \mathbb{R}^{n\times n}$ are the matrices of the local models and r is the number of local models of each subsystem $\sigma(x(t))$. It is assumed that the state of (1) does not jump at the switching instants, that is, the solution $x(t)$ is everywhere continuous. In this work, the dynamics behavior of (1) is investigated in the following subset of the state space:

$$Z := \{x(t) \in \mathbb{R}^n : |x_j(t)| \leq \bar{x}_j\}, \ j \in \mathcal{I} \tag{2}$$

where $\mathcal{I} \subset \{1, 2, \cdots, n\}$ and $\bar{x}_j$ is a known positive real number for all $j \in \mathcal{I}$. When convenient, the argument of function $h_{pk}(x(t))$ will be omitted and p will be used to represent the case $\sigma(x(t)) = p$.

From properties of membership functions the following relations hold:

$$\forall p \in \mathcal{P}, \ k \in \mathcal{R}, \ h_{pk} \geq 0, \ \ \sum_{k=1}^{r} h_{pk} = 1 \text{ and } \sum_{k=1}^{r} \dot{h}_{pk} = 0 \tag{3}$$

with $\mathcal{R} = \{1, 2, \cdots, r\}$.

Exploring (3), we can obtain

$$\left(\sum_{k\in\mathcal{R}} h_{\beta k} - \frac{1}{N-1} \sum_{\substack{p\in\mathcal{P} \\ p\neq\beta}} \sum_{k\in\mathcal{R}} h_{pk} \right) \left(\sum_{p\in\mathcal{P}} \sum_{k\in\mathcal{G}_p} h_{pk} \right) = 0 \tag{4}$$

with $\beta \in \mathcal{P}$ and $\mathcal{G}_p$ a subset of $\mathcal{R}$ for all $p \in \mathcal{P}$, which is previously chosen.

To establish the main results of this paper, consider the set

$$\mathscr{P} = \left\{ \alpha \in \mathbb{R}^N : \alpha_p \geq 0, \ \ \forall p \in \mathcal{P} \ \text{ and } \ \sum_{p=1}^{N} \alpha_p = 1 \right\} \tag{5}$$

and define

$$\begin{aligned} Z_v &= \left\{x \in Z : e_v' x = \bar{x}_v\right\} \bigcup \left\{x \in Z : e_v' x = -\bar{x}_v\right\}, \forall v \in \{1, 2, \cdots, n\}, \\ \Omega_\ell &= \{x \in Z : V(x) < \ell\}, \\ \Omega_{a,\ell} &= \Omega_a - \Omega_\ell, \quad \ell < a, \\ \mathcal{D} &= \left\{ x \in Z : x' \left[\sum_{p\in\mathcal{P}} \sum_{k\in\mathcal{G}_p} \dot{h}_{pk}(x) P_{pk} \right] x > 0 \right\}. \end{aligned}$$

Proposition 2.1. *If there exists $\alpha \in \mathscr{P}$ such that system*

$$\dot{x}(t) = \sum_{p=1}^{N} \alpha_p \sum_{k=1}^{r} h_{pk} \mathbf{A}_{pk} x(t) \tag{6}$$

is asymptotically stable, then there exists a switching law that ensures the asymptotic stability of the switched TS fuzzy system (1).

Proof. The proof follows [11]. □

In the next section, the S-procedure is explored to obtain less conservative conditions than presented in [11].

3 Main Result

Consider a scalar function $V : Z \to \mathbb{R}$ given by:

$$V(x) = x' \mathbf{P}(h) x \tag{7}$$

where $\mathbf{P}(h) = \sum_{p\in\mathcal{P}} \sum_{k\in\mathcal{G}_p} h_{pk} \mathbf{P}_{pk}$. Using (7) the following result is obtained.

Theorem 3.1. *Consider system (1) in set Z and parameters $a < \min_{x\in\partial Z} \sum_{p\in\mathcal{P}} \sum_{k\in\mathcal{G}_p} h_{pk}$, $b_1 \geq \max_{x\in\partial Z} \sum_{p\in\mathcal{P}} \sum_{k\in\mathcal{G}_p} h_{pk}$ and $b \geq ||x||, \forall x \in Z$. If for given real numbers $\epsilon > 0$ and $\alpha \in M$, there exists a real number $\ell < a$ such that $\sup_{x\in\mathcal{D}} V(x) < \ell$ and matrices $\mathbf{P}_{pk} = \mathbf{P}_{pk}' \in \mathbb{R}^{n\times n}$, $\mathbf{M} \in \mathbb{R}^{2n\times 2n}$, $\mathbf{L}_{pk} \in \mathbb{R}^{n\times n}$, $\mathbf{R}_{pk} \in \mathbb{R}^{n\times n}$ satisfying (8)-(17), then, every solution $\varphi_\sigma(t, x_0)$ of* (1)*, with $x_0 \in \Omega_a$, possessing a mixed switching law $\sigma(t, x)$ which attracts $x(t)$ to $\bar{\Omega}_\ell$ while $x(t) \in \Omega_{a,\ell}$.*

$$\Upsilon_{\beta k\_\beta k} + \mathbf{Q} \prec \mathbf{0}, \quad k \in \mathcal{G}_\beta \tag{8}$$

$$\Upsilon_{\beta k\_ij} + \mathbf{Q} \prec \mathbf{0}, \quad k \in \mathcal{R} - \mathcal{G}_\beta, \quad i \in \mathcal{P}, \quad j \in \mathcal{G}_i \tag{9}$$

$$\Upsilon_{\beta k\_\beta j} + \Upsilon_{\beta j\_\beta k} + 2\mathbf{Q} \prec \mathbf{0}, \quad j, k \in \mathcal{G}_\beta, \quad j < k \tag{10}$$

$$\Upsilon_{pk\_pk} - \frac{1}{N-1}\mathbf{Q} \prec \mathbf{0}, \quad p \in \mathcal{P} - \{\beta\}, \quad k \in \mathcal{G}_p \tag{11}$$

$$\Upsilon_{pk\_ij} - \frac{1}{N-1}\mathbf{Q} \prec \mathbf{0}, \quad p \in \mathcal{P} - \{\beta\}, \quad k \in \mathcal{R} - \mathcal{G}_p, \quad i \in \mathcal{P}, \quad j \in \mathcal{G}_i \tag{12}$$

$$\Upsilon_{pk\_pj} + \Upsilon_{pj\_pk} - \frac{2}{N-1}\mathbf{Q} \prec \mathbf{0}, \quad p \in \mathcal{P} - \{\beta\}, \quad k, j \in \mathcal{G}_p, \quad j < k \tag{13}$$

$$\Upsilon_{pk\_ij} + \Upsilon_{ij\_pk} - \frac{2}{N-1}\mathbf{Q} \prec \mathbf{0}, \quad p, i \in \mathcal{P} - \{\beta\}, \quad p < i, \quad k \in \mathcal{G}_p, \quad j \in \mathcal{G}_i \tag{14}$$

$$\Upsilon_{\beta k\_ij} + \Upsilon_{ij\_\beta k} + \frac{N-2}{N-1}\mathbf{Q} \prec \mathbf{0}, \quad k \in \mathcal{G}_\beta, \quad i \in \mathcal{P} - \{\beta\}, \quad j \in \mathcal{G}_i \tag{15}$$

$$\frac{1}{\bar{x}_v^2} e_v e_v' < P_{pk}, \forall v \in \{1, 2, ..., n\}, p \in \mathcal{P}, k \in \mathcal{G}_p \tag{16}$$

$$P_{pk} \leq \mu I, \ \forall p \in \mathcal{P}, \ k \in \mathcal{G}_p, \tag{17}$$

with

$$\begin{aligned}
\mathbf{Q} &= \sum_{p \in \mathcal{P}} \sum_{k \in (\mathcal{R} - \mathcal{G}_p)} \Upsilon_{pk_pk} + \mathbf{M} \quad \textit{and} \\
\Upsilon_{pk_ij} &= \begin{bmatrix} \alpha_i(\mathbf{L}_{pk}\mathbf{A}_{ij} + \mathbf{A}_{ij}'\mathbf{L}_{pk}') + \epsilon \mathbf{P}_{pk}/N - \epsilon \frac{\ell I}{N b_1 b^2} & \star \\ \mathbf{P}_{pk}/N - \mathbf{L}_{pk}'/N + \alpha_i \mathbf{R}_{pk}\mathbf{A}_{ij} & (-\mathbf{R}_{pk} - \mathbf{R}_{pk}')/N \end{bmatrix}.
\end{aligned}$$

Proof. From [11], we have that the level set Ω_a is contained in Z and $\partial\Omega_a \cap \partial Z = \emptyset$. Now, multiplying (8) by $h_{\beta k}^2$, (9) by $h_{\beta k}h_{ij}$, (10) by $h_{\beta k}h_{\beta j}$, (11) by h_{pk}^2, (12) and (14) by $h_{pk}h_{ij}$, (13) by $h_{pk}h_{pj}$, (15) by $h_{\beta k}h_{ij}$ and adding all terms we obtain:

$$\begin{aligned}
&\sum_{k \in (\mathcal{R} - \mathcal{G}_\beta)} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{G}_i} h_{\beta k} h_{ij} \left(\Upsilon_{\beta k_ij} + \mathbf{Q}\right) + \sum_{k \in \mathcal{G}_\beta} \sum_{\substack{j \in \mathcal{G}_\beta \\ j<k}} h_{\beta k} h_{\beta j} (\Upsilon_{\beta k_\beta j} + \Upsilon_{\beta j_\beta k} + 2\mathbf{Q}) \\
&+ \sum_{\substack{p \in \mathcal{P} \\ p \neq \beta}} \sum_{k \in \mathcal{G}_p} h_{pk}^2 \left(\Upsilon_{pk_pk} - \frac{1}{N-1}\mathbf{Q}\right) + \sum_{\substack{p \in \mathcal{P} \\ p \neq \beta}} \sum_{k \in (\mathcal{R} - \mathcal{G}_p)} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{G}_i} h_{pk} h_{ij} \left(\Upsilon_{pk_ij} - \frac{1}{N-1}\mathbf{Q}\right) \\
&+ \sum_{k \in \mathcal{G}_\beta} h_{\beta k}^2 \left(\Upsilon_{\beta k_\beta k} + \mathbf{Q}\right) + \sum_{\substack{p \in \mathcal{P} \\ p \neq \beta}} \sum_{k \in \mathcal{G}_p} \sum_{\substack{i \in \mathcal{P} \\ i \neq \beta \\ i > p}} \sum_{j \in \mathcal{G}_i} h_{pk} h_{ij} \left(\Upsilon_{pk_ij} + \Upsilon_{ij_pk} - \frac{2}{N-1}\mathbf{Q}\right) \\
&+ \sum_{\substack{p \in \mathcal{P} \\ p \neq \beta}} \sum_{k \in \mathcal{G}_p} \sum_{\substack{j \in \mathcal{G}_p \\ j<k}} h_{pk} h_{pj} \left(\Upsilon_{pk_pj} + \Upsilon_{pj_pk} - \frac{2}{N-1}\mathbf{Q}\right) \\
&+ \sum_{k \in \mathcal{G}_\beta} \sum_{\substack{i \in \mathcal{P} \\ i \neq \beta}} \sum_{j \in \mathcal{G}_i} h_{\beta k} h_{ij} \left(\Upsilon_{\beta k_ij} + \Upsilon_{ij_\beta k} + \frac{N-2}{N-1}\mathbf{Q}\right)
\end{aligned}$$

$$\begin{aligned}
&= \sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{G}_p} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{R}} h_{pk} h_{ij} \Upsilon_{pk_ij} \\
&+ \left(\sum_{k \in \mathcal{R}} h_{\beta k} - \frac{1}{N-1} \sum_{\substack{p \in \mathcal{P} \\ p \neq \beta}} \sum_{k \in \mathcal{R}} h_{pk}\right) \left(\sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{G}_p} h_{pk}\right) \mathbf{Q} \prec \mathbf{0}.
\end{aligned} \tag{18}$$

Replacing (4) in (18) we have that

$$\begin{bmatrix} \mathbf{L}(h)\mathbf{A}(\alpha,h)+\mathbf{A}(\alpha,h)'\mathbf{L}(h)'+\epsilon P(h)-\epsilon\frac{\ell I}{b_1b^2}\sum_{p\in\mathcal{P}}\sum_{k\mathcal{G}_p}h_{pk} & \star \\ \mathbf{P}(h)-\mathbf{L}(h)'+\mathbf{R}(h)\mathbf{A}(\alpha,h) & -\mathbf{R}(h)-\mathbf{R}(h)' \end{bmatrix} \prec \mathbf{0} \tag{19}$$

where $\mathbf{L}(h)=\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}\mathbf{L}_{pk}$, $\mathbf{R}(h)=\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}\mathbf{R}_{pk}$ and $\mathbf{A}(\alpha,h)=\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{R}}\alpha_p h_{pk}\mathbf{A}_{pk}$.
Pre-multiplying and post-multiplying (19) by the vector $[x' \;\; x'\mathbf{A}(\alpha,h)']$ and its transpose, respectively, it yields

$$x'\left\{\mathbf{A}(\alpha,h)'\mathbf{P}(h)+\mathbf{P}(h)\mathbf{A}(\alpha,h)+\epsilon\mathbf{P}(h)-\epsilon\frac{\ell I}{b_1b^2}\sum_{p\in\mathcal{P}}\sum_{k\mathcal{G}_p}h_{pk}\right\}x \prec \mathbf{0}, \tag{20}$$

that is,

$$\begin{aligned} x'\{\mathbf{A}(\alpha,h)'\mathbf{P}(h)+\mathbf{P}(h)\mathbf{A}(\alpha,h)\}x &< -\epsilon V(x)+\epsilon\frac{\ell||x||^2}{b_1b^2}\sum_{p\in\mathcal{P}}\sum_{k\mathcal{G}_p}h_{pk} \\ &\leq -\epsilon(V(x)-\ell)<0, \end{aligned} \tag{21}$$

for all x such that $V(x)>\ell$.

The time-derivative of the function (7) along the trajectories of (1) is given by:

$$\dot{V}(x)=x'\left\{\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}\dot{h}_{pk}\mathbf{P}_{pk}\right\}x+x'\left\{\mathbf{A}(\alpha,h)\mathbf{P}(h)+\mathbf{P}(h)\mathbf{A}(\alpha,h)\right\}x. \tag{22}$$

By (21), we can say that the second part of (22) is negative definite whenever $x\notin\Omega_\ell$. Since $\sup_{x\in\mathcal{D}}V(x)<\ell<a$, we can conclude by Lemma 1 from [11] that every solution $\varphi_\sigma(t,x_0)$ of (1) with $x_0\in\Omega_a$ possessing a measurable mixed switching law $\sigma(t,x)$ is attracted to $\bar{\Omega}_\ell$ while $x(t)\in\Omega_{a,\ell}$. Moreover, if (16) and (17) holds, then $x'\sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}P_{pk}x \leq \sum_{p\in\mathcal{P}}\sum_{k\in\mathcal{G}_p}h_{pk}x'\mu x$. Therefore, minimizing μ the set $\{x\in\mathbb{Z}: x'x\leq\frac{a}{b_1\mu}\}\subseteq\Omega_a$, makes Ω_a to be maximized. □

Remark 3.1. *From [11], a measurable mixed switching law $\sigma(t,x)$ satisfying Theorem 3.1 can obtained with*

$$\sigma(x)=\begin{cases} 1, & \textit{if} \quad x\in\Gamma_1 \\ p, & \textit{if} \quad x\in(\Gamma_p\setminus(\bigcup_{k<p}\Gamma_k)) \end{cases} \tag{23}$$

where $\Gamma_p=\{x\in\mathcal{B}:\nabla V(x)f_p(x)<0$ and $\nabla V(x)f_p(x)\leq\nabla V(x)f_k(x),\ \forall k\in\mathcal{P}-\{p\}\}$.

In the next example presents a comparison between the feasibility region obtained with Theorem 1 from [11] and Theorem 3.1.

Example

Consider a switched TS fuzzy system (1) with the following local models:

$$\mathbf{A}_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{A}_{21} = \begin{bmatrix} -10 & a_1 \\ 0 & a_2 \end{bmatrix}, \quad \mathbf{A}_{22} = \begin{bmatrix} -10 & -15 \\ 0 & -3 \end{bmatrix}, \qquad (24)$$

and membership functions

$$h_{11} = \frac{50 - x_1^2 - x_2^2}{50}, \quad h_{12} = 1 - h_{11}; \quad h_{21} = \frac{x_2^2}{25}, \quad h_{22} = 1 - h_{21} \qquad (25)$$

in the set $Z = \{x \in \mathbb{R}^2 : |x_1| \leq 5 \ \text{ and } \ |x_2| \leq 5\}$. Adopting parameters $b = 2\sqrt{5}, b_1 = 1.5, \alpha_1 = 0.6, \alpha_2 = 0.4, \beta = 1, \mu = 0.19, \epsilon = 0.7$ and $\ell = 0.2$. Figure 1 shows the feasible region in the plane $a_1 \times a_2$ obtained with [11] and Theorem 3.1. Note that the conservatism is reduced significantly by Theorem 3.1.

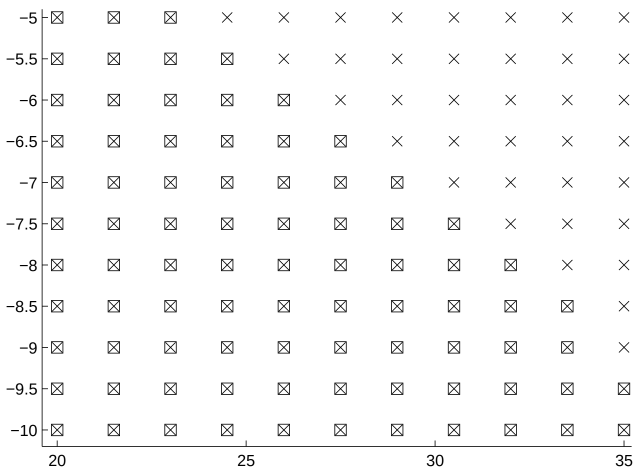

Figure 1: Comparison of the region of the plane $a_1 \times a_2$ between [11] (□) and Theorem 3.1 (×).

4 Conclusions

In this work, LMI-Based conditions to ensure the ultimate boundedness of the solutions of switched TS fuzzy systems were proposed. The result provided sufficient conditions to the existence of an auxiliary function $V(x)$, such that, its time derivative can be positive in some bounded sets. Finally, the S-procedure was used to obtain less conservative conditions than recent published results. A numerical example illustrates the efficacy of the proposed method.

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