Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 7, n. 1, 2020.

Trabalho apresentado no XXXIX CNMAC, Uberlândia - MG, 2019.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Methane Dispersion in Wetland Areas of Mato Grosso – Brazil

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Abstract. In this paper, we propose a mathematical model that represents the process of methane dispersion, produced by the decomposition of organic matter by bacteria in anaerobic media, such as flooded areas. Because the methane is one of the greenhouse gases that contributes about 20% of the causes of global warming, this study will contribute to understanding the process that occurs in the lower atmosphere and what actions can be taken to minimize the problem. For a better visualization, the model was implemented through the finite element method, via Galerkin method, for spatial discretization and by Crank-Nicolson for temporal discretization, through a numerical code written in Matlab^{TM} environment, which made it possible to generate animations to the evolutionary process of dispersion of methane into the atmosphere.

Keywords. Mathematical Modeling, Greenhouse Gases, Finite Elements Method (FEM).

1 Introduction

Methane is produced from complex biochemical reactions of the anaerobic decomposition of organic materials. This process occurs on a large scale where the decomposition of plants submerged in water, for example, in marshes and swamps, and in moist soils destined for the rhizicultura. Wetlands are the largest natural source of methane emissions as well as large hydroelectric lakes, especially when the wood is not previously removed from the area to be flooded [1].

According to the 2007 IPCC Report, methane concentration in the global atmosphere more than doubled from pre-industrial times, rising from 715 ppb (parts per billion) to 1732 ppb in the early 1990s. It is assumed that the increase in the concentration of methane in the atmosphere is largely anthropogenic, by activities such as increase food production, intense use of fossil fuels and deforestation [2].

Over the past three decades, variations in emissions from wetlands have dominated the variability of surface emissions from year to year, and that flooded areas contribute most of the natural emissions (about 32% of the total emitted).

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The Pecus Pantanal (Embrapa Pantanal) project is measuring the emissions of enteric methane (produced by enteric fermentation of cattle, which is a natural process of ruminant digestion) and methane emissions from soil of floodplain areas. This emission of soil is a process that is also natural, common in regions of floodplains. The gas is produced in soaked soils, with little oxygen.

Mathematical modeling attempts to reproduce the relevant characteristics of environmental phenomena for the treatment of a particular specific issue. There are several methodologies and models that describe this natural phenomenon and, generally, the models are given through partial differential equations (PDE) [7, 8]. Such equations, almost always, when they have a solution, can not be expressed in terms of elementary functions. With the aid of computational resources, through numerical simulation, we can analyze the phenomena in order to try to explain the changes in state variables, in the concentration of methane in the atmosphere.

The equations that represent the phenomenon under study are discretized numerical method [5]. The traditional methods available for the treatment of these differential equations are the Finite Differences Method, the Finite Volume Method and the Finite Elements Method, the latter being the method used in this paper.

2 The Study Area

Located in the center of South America, the Pantanal biome is considered one of the largest continuous wetlands on the planet. Its area is 150,355 km^2, thus occupying 1.76% of the total area of the Brazilian territory, with 65% of its territory in the state of Mato Grosso do Sul and 35% in Mato Grosso.

In Figure 1 we present the area of study, including the five main bays of the region.

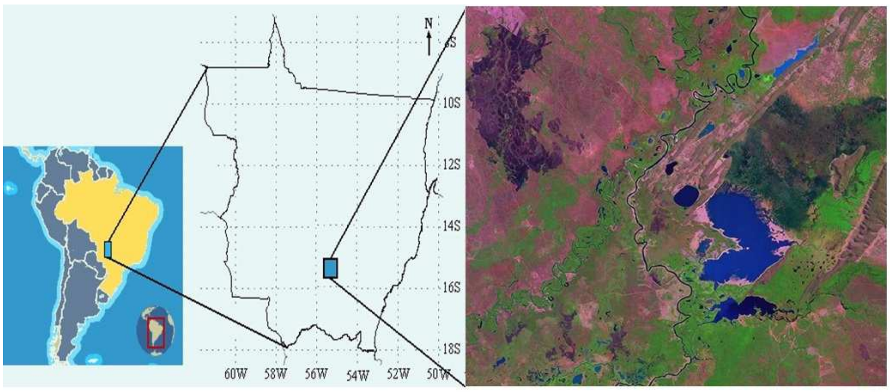

Figure 1: Maps for location of the study area: Pantanal Matogrossense, Brazil.

This continental biome is considered the one with the smallest territorial extension in Brazil. However, this fact does not detract from the exuberant wealth that the biome referent houses. In its territorial space the biome, that is an alluvial plain, is influenced by rivers that drain the basin of the Upper Paraguay. Despite its exuberant natural beauty

DOI: 10.5540/03.2020.007.01.0398

the biome has been greatly impacted by human action, mainly by agricultural activity, especially in the areas adjacent to the biome plateau. The area of flood in the region ranges from 11,000 to 110,000 km^2, depending on the year, with a historical average of 53,000 km^2.

In this way, large areas can remain submerged by flood, due to the overflow of rivers or flooding resulting from local rains and raising the level of the water table for up to 8 months. Rain and dry seasons are well defined.

The volume of rainfall in summer is much higher than in winter. Therefore, summer is a rainy season in the Pantanal, while winter is dry. The index rainfall of the Pantanal is 1,110 mm per year. Relative humidity is around 50% in winter and 75% in summer [3].

3 The Mathematical Model

The mathematical model we use to describe the phenomenon of methane dispersion in the lower atmosphere is through the classical diffusion-advection equation given by

$$\frac{\partial u}{\partial t} = -\nabla\left(-\alpha\nabla u\right) - \nabla\left(\vec{\mathcal{V}}\cdot u\right) - \sigma u + f \tag{1}$$

where

- $u(x,y,t)$ represents the concentration of methane in the point (x,y) of the two-dimensional domain Ω for a instant $t \in (0,T]$;
- α is the effective diffusion coefficient in the air;
- $\vec{\mathcal{V}}$ represents the field of winds that makes the transport in the air means;
- σ is the global decay coefficient, and
- f represent the term source of methane.

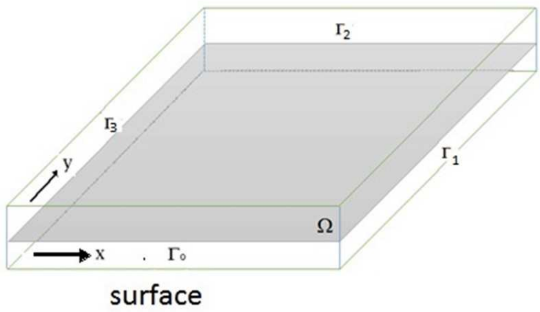

Figure 2: The domain of study area.

In addition, the boundary conditions (see Figure 2) considered in this problem are of the Robin type, given by

$$-\alpha\left.\frac{\partial u}{\partial \eta}\right|_{\Gamma_0\cup\Gamma_1} = 0 \qquad -\alpha\left.\frac{\partial u}{\partial \eta}\right|_{\Gamma_2\cup\Gamma_3} = \beta u \tag{2}$$

with β representing the permeability at the boundary $\Gamma_2 \cup \Gamma_3$.

Finally, the initial condition is given by

$$u(x,y,0) = u_0(x,y) \tag{3}$$

The equations (1)–(3) are called the classical formulation of the problem.

4 The Weak Formulation

In order to obtain the numerical approximations for solution of the equations (1)–(3), using variational techniques on appropriate spaces, we obtained the weak formulation, which allows to obtain in a simpler form guarantee existence and uniqueness of solution for the weak formulation, as well as facilitates of adoption of the finite element method for computational implementation.

In this sense, the weak equivalent formulation for the problem in its classical (or strong) formulation becomes

$$\begin{aligned}\iint_{0;\Omega} \frac{\partial u}{\partial t}\nu d\mu = \iint_{0;\Omega} &\left[\alpha\left(\nabla u \cdot \nabla \nu\right) - \mathcal{V}_x \nabla u \nu - \mathcal{V}_y \nabla u \nu - \sigma u \nu + f(x,y)\nu\right] d\mu \\ &+ \int_{\Gamma_2 \cup \Gamma_3} \beta u \nu d\gamma\end{aligned} \tag{4}$$

In Figure 3, we present the discretized domain by elements of first order, which allowed the implementation of numerical codes in Matlab$^{\mathrm{TM}}$ environment for the computational simulations.

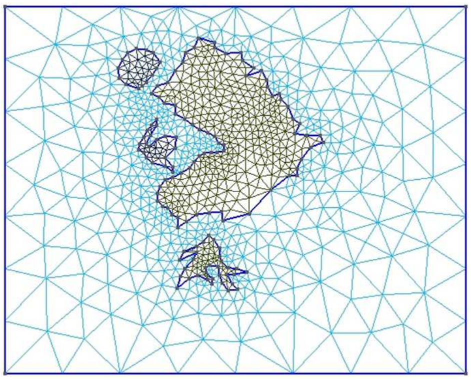

Figure 3: The discretization of the domain Ω.

5 Results

In order to obtain the computational simulations of scenarios by numerical approximations of solution for the equation (4), we consider a periodic function $f(x,y,t)$ for the source term, given by eq. (5)

$$f(x,y,t) = \begin{cases} K + \rho\cos(\omega t) & \text{if} \quad (x,y) \text{ is on the water surface;} \\ 0 & \text{if} \quad (x,y) \text{ is on the land surface,} \end{cases} \tag{5}$$

where K is the average charge of methane outflow of the water surface, ρ is the amplitude of oscillation of the methane concentration that is released throughout the day, and ω is the period of the cycle, in this case $\frac{2\pi}{24} = \frac{\pi}{12}$.

We adopted the value for the diffusion coefficient (α) based on [4], and the value of the global decay coefficient (σ) was empirical.

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The parameters used for the first simulation are presented in the following Table 1.

Table 1: Values of parameters used in this simulation.

Parameter	Symbol	Value	Unit
Velocity of wind	$\mathcal{V}$	3.2	km/h
Direction of wind	θ	$\pi/4$	radians
Source of methane	K	0.018	mol/km^2/h
Amplitude of oscillation	ρ	0.028	mol/km^2/h
Period of cycle	ω	$\pi/12$	radians/h
Permeability in the border	β	0.1	km/h
Decay coefficient	σ	0.015	km^2/h
Diffusion coefficient	α	0.0012	km^2/h

In Figure 4, we present the concentration of methane for the four chosen nodes of the domain along of 20 days, where is possible to verify the difference of concentration for each node in the study area along of time, depending on the position in the domain.

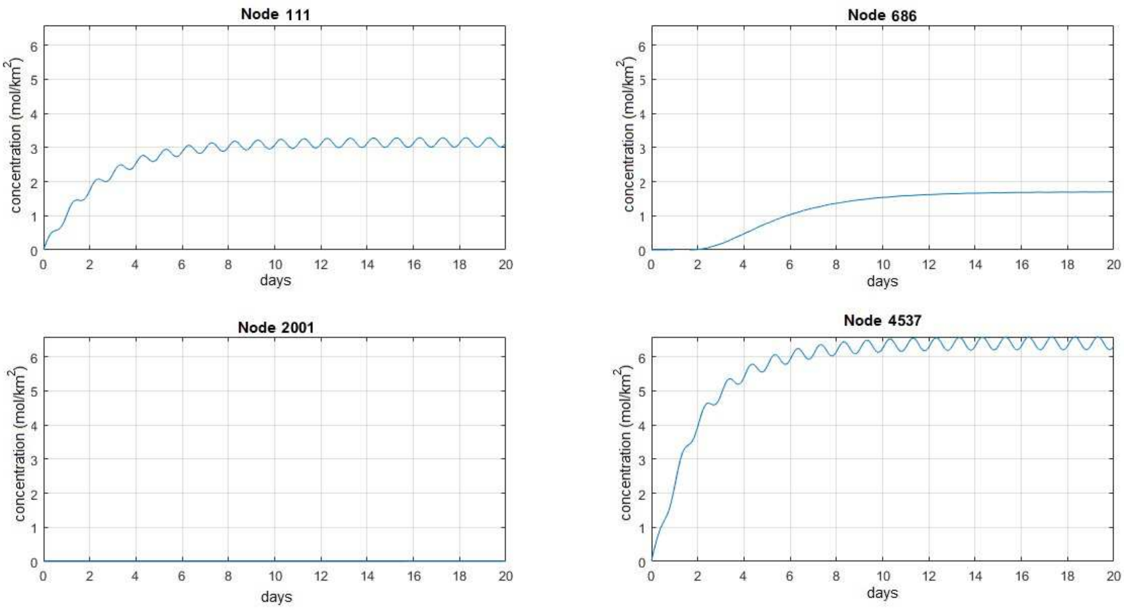

Figure 4: The evolutionary concentration of methane for four chosen nodes.

In Figure 5, we present the concentration of methane for the same four chosen nodes of the domain along of 20 days, the difference of concentration for each node is also observed in the study area, depending on the position in the domain. In addition, the concentration on the node 2001 increase significantly, in this case the velocity of wind considered was higher than the previous simulation. In this simulation we adopted the wind velocity as $V = 10$ km/h, and the direction of wind as $\theta = \pi/4$ radians, but the others parameters were the same.

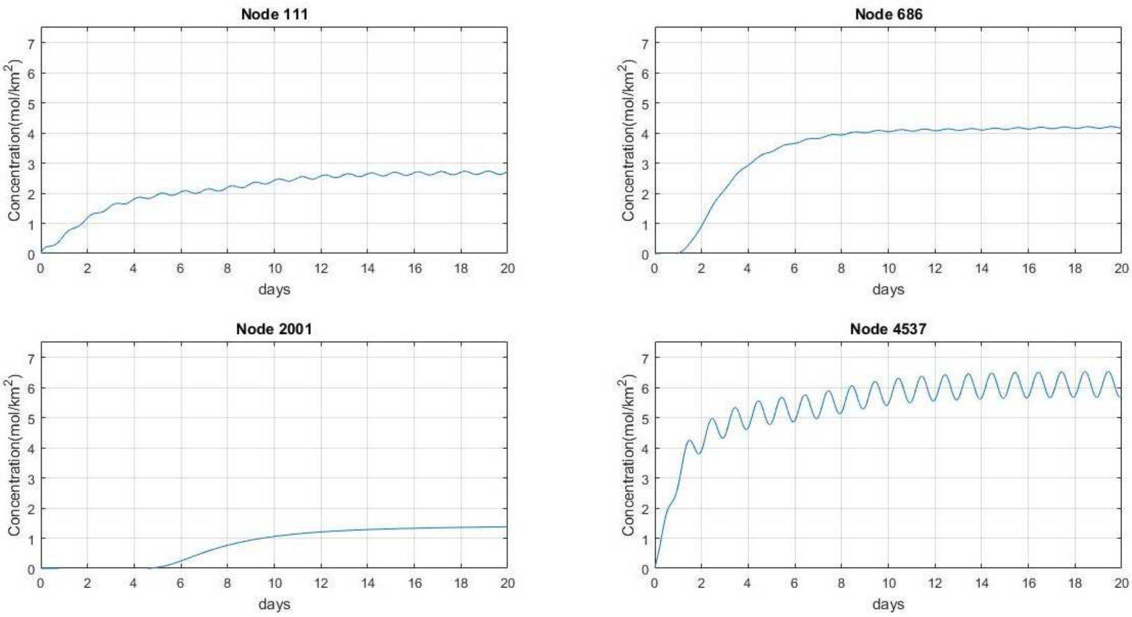

Figure 5: The evolutionary concentration of methane for four chosen nodes.

6 Conclusion

The essential contribution of this paper is the function adopted to describe the methane source term, in order to consider the observations presented by [6], in which he presented a model of methane flux from the Florida Everglades, and that the production methane is seasonal, with higher production in the summer and lower production in the winter season. Thus, we adopt for the local condition with a cycle throughout the day, with higher diurnal production and lower nocturnal production.

In addition, the numerical code created and implemented for the simulations presented qualitatively satisfactory results, although there are not yet data collected for their validation, proved to be efficient and easy to adapt to the other study regions.

In this sense, the mathematical model considered in this paper makes a significant contribution to the elaboration of experimental design and data collection, which can contribute in an effective way to the study of methane gas dispersion in the atmosphere and its consequences for the local micro-climate.

7 Acknowledgement

The first author thanks the support of the FAPEMAT by the research fellowship (Ed.002/2014).

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DOI: 10.5540/03.2020.007.01.0398