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Assembling of metamaterials beams with uncertain scattering properties

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Resumo. Metamaterials, or locally resonant metamaterials, are a class of structures that have been used to control and to manipulate acoustic and elastic waves with applications in vibration attenuation. A great amount of research has been done on acoustic and structural metamaterials but very few attention has been given to the effects of coupling conditions on structural assemblies, even though this is typical case on mechanical engineering applications. In this work, the wave attenuation in a metamaterial beam assembly is investigated considering uncertain connections. A beam, with attached resonators, undergoing longitudinal and flexural vibration is connected to homogeneous beams at each end. It is assumed a large enough number of identical resonators such that effective longitudinal and flexural wavenumbers are derived. Wave modes are assumed unchanged by the attachments and analytical expressions can be derived. A point connection is considered with an assembly angle such that wave mode conversion, between flexural and longitudinal waves, can happen. The reflection and transmission properties of the assemble are then calculated and it is shown that the angle of the assembly has a significant effect on the band gap performance. The uncertainty analysis focus on the variability of the connection angles and ensemble statistics are investigated.

Palavras-chave. Metamaterial, band gap, structural assembly, stochastic modelling

1 Introduction

Metamaterials, or locally resonant metamaterials, are a class of structures that have been used to control and to manipulate acoustic and elastic waves [6] with applications in vibration attenuation [5]. Although periodicity of the resonators positioning is not required, it is used for a cell-based description of the wave propagation. In metamaterials.,

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the attenuation effect is created due to inclusions or attachments that work as internal resonators [7] and are able to create band gaps at sub-wavelength frequencies, unlike the phononic crystals, which rely on spatial periodicity and the Bragg scattering effect [6].

In this work, the wave attenuation from a metamaterial beam assembly is investigated considering parametric uncertainties. A metamaterial beam undergoing longitudinal and flexural vibration is considered and it is connected to a simple bare beam at each end. A point connection is considered with a angle such that wave mode conversion between flexural and longitudinal waves can happen. The scattering properties of the assemble can then be calculated and it is shown that the angle of the assembly has a significant effect on the band gap performance. The uncertainty analysis focus on the variability of the connection angles and ensemble statistics are investigated. Two cases are considered, one which only one connection angle is uncertain and other which both angles are uncertain. For each case, two probabilistic models are assumed and it is shown that the choice stochastic model increases the variability of the results. It is shown that introducing connection angles between the metamaterial and the host structure can significantly increases the attenuation performance due to the wave mode conversion at the joints. Moreover, the choice of sets of random variable played a much more important role in the results than the probabilistic models for the random variables.

1.1 Wave model

The equation of motion of a continuous system with S periodically attached resonators can be given in the general form by [8]

$$L(x)w(x,t) + \mu\ddot{w}(x,t) - \sum_{p=1}^{S} k_p u_p(t)\delta(x - x_p) = p(x,t), \tag{1}$$

and one additional equation for each resonator $m_p\ddot{u}_p(t) + k_p u_p(t) + m_p\ddot{w}(x_p,t) = 0$, where $u_p(t)$ is the displacement of each resonator attached at x_p, with mass m_p and stiffness k_p and $\delta(x)$ is the Dirac delta function. This expression was originally proposed for a modal analysis in metastructures and allows the derivation of closed form expression for the band gap frequency edges. In this work, it will be used for finding the dispersion equation. Also, assuming that the wave modes are unchanged due to the resonators attachments, it provides a analytical framework for calculating reflection and transmission coefficients. Assuming identical resonators and a large enough number of attachments, it can be shown that

$$L(-ik) - \mu\omega^2\left(1 + \epsilon\frac{1}{1 - \Omega_r^2}\right) = 0, \tag{2}$$

where $\Omega_r = \omega/\omega_r$ and $\omega_r^2 = k_p/m_p$ and $\epsilon = m_p/\mu\Delta l$ is the mass ratio for resonators spaced by Δl. The suitable stiffness operators can be applied to find the effective wavenumbers for longitudinal and flexural waves

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$$k_{rl} = \sqrt{\frac{\rho}{E}\left(1+\epsilon\frac{1}{1-\Omega_r^2}\right)}\omega, \quad k_{rb} = \sqrt[4]{\frac{\rho A}{EI}\left(1+\epsilon\frac{1}{1-\Omega_r^2}\right)}\sqrt{\omega}. \tag{3}$$

This result is equivalent to [3] for a continuous neutralizer attached to the beam, in which the mass ratio is given in terms of wave length. Assuming that the attached resonators do not change the wave types, these wavenumbers can then be used to describe the displacement field the same as for the simple beam.

1.2 Metamaterial assembly

A metamaterial beam undergoing longitudinal and flexural waves is connected to two other homogeneous beam, one at each end, as shown in Fig. 1. At the left end, the connection angle is α_1, $\mathbf{b}_1^{\pm}$ are the amplitude of the incoming and outgoing waves. At the right end, the connection angle is α_2, $\mathbf{a}_1^{\pm}$ are the amplitude of the outgoing and incoming waves. A scattering matrix can be defined relating the incoming and outgoing waves of the assembly by [2, 4]

$$\begin{bmatrix} \mathbf{a}_2^+ \\ \mathbf{b}_1^- \end{bmatrix} = \begin{bmatrix} \mathbf{r}^+ & \mathbf{t}^+ \\ \mathbf{t}^- & \mathbf{r}^- \end{bmatrix} \begin{bmatrix} \mathbf{a}_2^- \\ \mathbf{b}_1^+ \end{bmatrix}, \tag{4}$$

where $\mathbf{r}^{\pm}$ are reflection matrices and $\mathbf{t}^{\pm}$ are transmission matrices. They can be obtained from the equilibrium and continuity conditions at the beams connections and the wave propagation along the metamaterial beam. Assuming $\mathbf{a}_2^- = \mathbf{0}$, i.e., a incident wave on the left end only, then the scattering simplifies to $\mathbf{a}_2^+ = \mathbf{t}^+\mathbf{b}_1^+$ and $\mathbf{b}_1^- = \mathbf{r}^-\mathbf{b}_1^+$. Therefore, the transmission coefficient $\mathbf{t}^+$ can be used as to access the vibration attenuation of the metamaterial beam in the assembly.

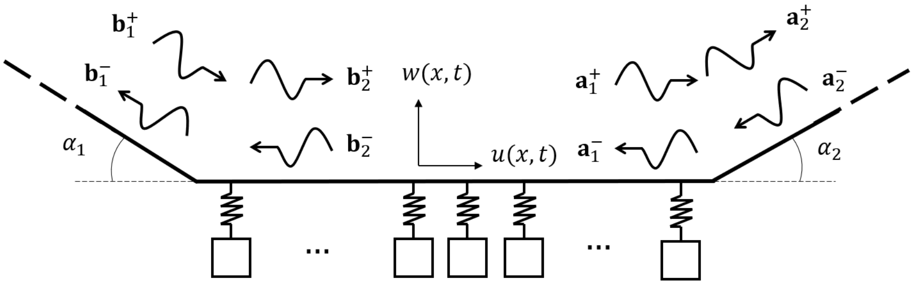

Figure 1: Metamaterial beam assembly with one semi-infinite homogeneous beam at each end undergoing longitudinal and flexural vibration.

In this case, the reflection and transmission matrices are size 3×3 and relate the longitudinal, propagating flexural and non-propagating flexural (near field) wave amplitudes at the both sides of the assembly. For $\alpha_1 = \alpha_2 = 0$, i.e., a straight assembly, no wave mode conversion is expected and these matrices are diagonal. However, for $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$, they are full matrices and wave mode conversion plays a role on the metamaterial vibration attenuation performance. Moreover, asymmetries in the assembly can be given by differences in the connection angle, i.e. $\alpha_1 \neq \alpha_2$, and also play a role on the reflection coefficients $\mathbf{r}^{\pm}$, while $\mathbf{t}^{+} = \mathbf{t}^{-}$ due to reciprocity.

2 Probabilistic modelling

Two cases are considered in the analysis. In the first, it defined that the first connection angle α_1 is fixed while $\alpha_2 = \alpha_1 + \theta$, where θ is a sample of the random variable Θ. In the second case, it is considered that both connection angles α_1 and α_2 can be modelled by the random variables $\mathtt{A}_1$ and $\mathtt{A}_2$, respectively. For each analysis case, a probabilistic model is defined. The probability distribution of the random variables have to take in to account physical constrains of the problem. Typically, manufacturing processes can only guarantee minimum θ_1 and maximum θ_2 values from the tolerances in the assembly process. It is also reasonable to assume that the angles in both connections are not correlated. Given the lack of any additional information, it is reasonable to assumed a the uniform PDF, i.e.

$$f^{(1)}_{\Theta,\mathtt{A}_1,\mathtt{A}_2}(x) = \frac{1}{\theta_2 - \theta_1}\,, \quad \theta_1 \leq x \leq \theta_2, \tag{5}$$

where θ_1 and θ_2 are the lower and the upper limits of the random variable Θ.

3 Numerical results

In this section, numerical results are presented considering the metamaterial beam assembly. All of the beams with and without resonators are composed of polyamide, whose mechanical properties are described in [1]. The metamaterial beam is 20 cm long and the resonators have a flexural natural frequency at 900 Hz and longitudinal natural frequency at 1300 Hz. Figure 2 presents the real and imaginary part of the longitudinal and flexural wavenumber for the bare beam and the absolute value of the transmission coefficient, considering $\alpha_1 = \alpha_2 = 0$, i.e., a straight assembly. For a lossless waveguide, the wavenumber can be real, leading to a propagating wave, imaginary, giving a decaying or evanescent wave, or complex, which has both behaviours, i.e. propagating and decaying. The imaginary part of the dispersion curve (negative values) shows the frequency band in which there is vibration attenuation for each wave mode, i.e. the band gap for longitudinal and flexural waves. Note that the wave types do not interact because the axial and flexural vibration are considered uncoupled at the metamaterial beam. This is also noticed in the absolute value of the transmission coefficient, which shows a very low transmission at the band gap frequencies for each individual wave mode. Additionally, from the dispersion curve it can be seen that the group velocity $c_g = \partial\omega/\partial k$, that gives the velocity of energy transport, is zero at the resonator frequency and it is negative at the band gap.

The effects of uncertainties on α_1 and α_2 at the wave mode conversion and the band gap performance are also investigated. For both considered cases, i.e. models with random variables Θ and $\mathtt{A}_1$ and $\mathtt{A}_2$, the probabilistic model considered that $\alpha_1 = \alpha_2 = 0$, with $\theta_1 = -\pi/10$ and $\theta_2 = \pi/10$. For the stochastic analysis, 1000 MC samples are used which is enough for convergence.

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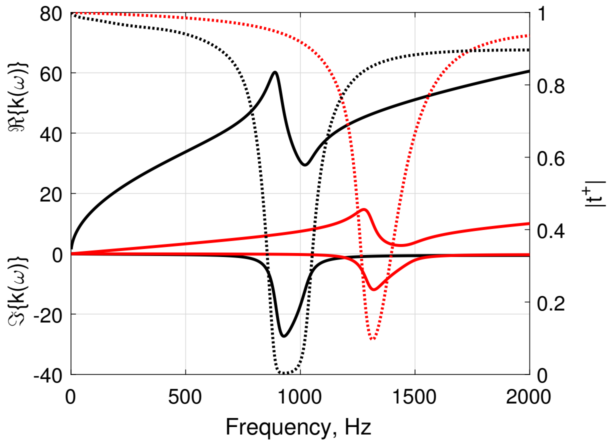

Figure 2: (left axis) Real and imaginary parts of the longitudinal (red) and flexural (black) wavenumbers (solid line) for the bare beam and metamaterial beam and (right axis) absolute value of the transmission coefficient (dashed line) considering $\alpha_1 = \alpha_2 = 0$.

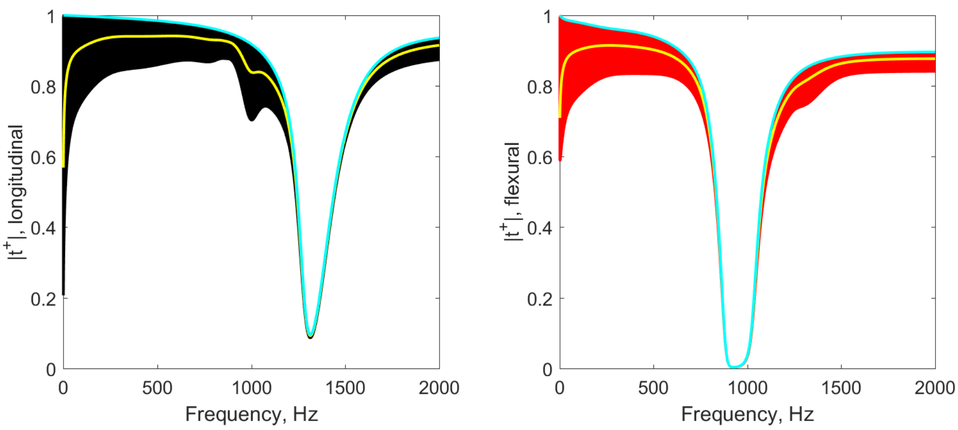

Figure 3: Mean (yellow), nominal (cyan) value and MC samples of the absolute value of the transmission coefficient considering $\alpha_1 = \alpha_2 = 0$ and Θ. Uniform PDF.

Figures 3 and 4 present the mean, nominal value and MC samples of the absolute value of the transmission coefficient obtained. It can be noticed that the mean value and the nominal response are not equivalent in all of the frequency band but at the band gap regions for each longitudinal or flexural wave modes. Therefore, the deterministic analysis is not representative of the typical behaviour of the transmission coefficient outside of this regions. In fact, the results show that the nominal response gives the upper bounds of the MC samples outside the band gap regions in both cases, while it is representative of the mean response in the band gap regions. The nominal model cannot capture the wave mode conversion occurring due to the random variation of the connection angles and it cannot predict the improved attenuation features observed in these cases. Moreover, the

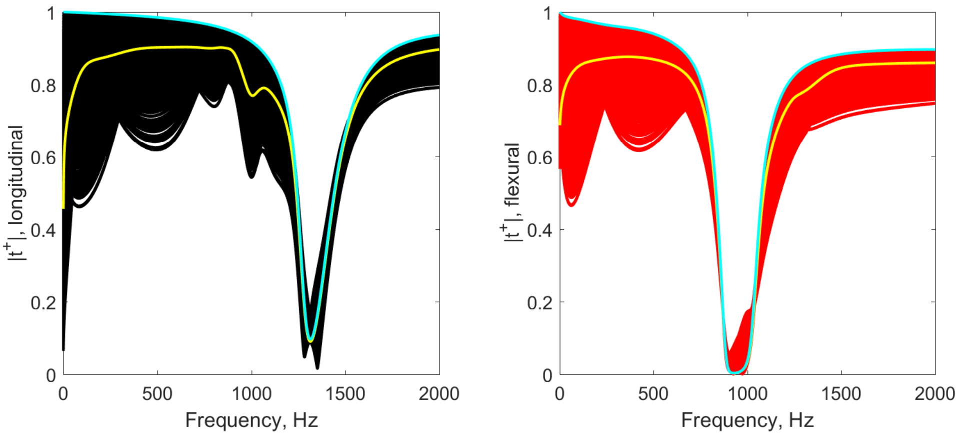

Figure 4: Mean (yellow), nominal (cyan) value and MC samples of the absolute value of the transmission coefficient considering the random variable of both connection angles, $\mathtt{A}_1$ and $\mathtt{A}_2$. Uniform PDF.

choice of sets of random variable played a much more important role in the results than the probabilistic models for the random variables. Note that changing from uniform to a truncated Gaussian slightly affects the mean values and the tails of the distribution of the results. The model considering both connection angles and uncertainty introduced qualitative changes on the response, with frequency bands with increased attenuation performance. This is because the wave mode conversion between longitudinal and flexural waves could occur at both connections.

4 Concluding remarks

The wave attenuation performance of a metamaterial beam assembly is investigated considering uncertain connections. It is assumed a large enough number of identical resonators such that and effective longitudinal and flexural wavenumbers are derived. Wave modes are assumed unchanged by the attachments and then analytical expressions can be derived. The reflection and transmission properties of the assembly are be calculated and it is shown that the angle of the assembly has a significant effect on the band gap performance.

The uncertainty analysis focus on the variability of the connection angles and ensemble statistics are investigated. Monte Carlo sampling is used as the stochastic solver. It is shown that the deterministic analysis is not representative of the typical behaviour of the transmission coefficient outside the band gap region. In this case, the nominal response gives the upper bounds of the MC samples outside the band gap regions in both cases, while it is representative of the mean response in the band gap regions.

Most importantly, it is shown that the nominal model, which does not include variability in the connections, cannot capture the wave mode conversion occurring due to the randomness of the connection angles and it cannot predict the improved attenuation features observed in these cases. Moreover, the choice of sets of random variable played a much more important role in the results than the probabilistic models for the random variables.

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