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Vibration Attenuation with Random Shunted Piezo Patches

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Abstract. This work deals with passive reduction of structural vibration by means of random shunted piezoelectric patches. The systems analyzed, shunted structures, are composed of a mechanical system with piezoelectric elements shunted with LR circuits. They are electromechanical systems, i.e, coupled systems. Due to the coupling, energy can be transferred between the mechanical and electromagnetic parts of the system. Tuning the electromechanical parameters, it is possible to control the energy flow between the mechanical and electromagnetic parts in a way that the amplitude of vibration of the structure is attenuated over a range of excitation frequency. The idea is to use the shunted piezoelectric patches as dampers to the structure vibration. When the shunt is perfectly tuned to the resonance frequency to control, the vibration attenuation is optimal. However, the damping performance is subjected to uncertainties in the electromechanical parameters. The objective of the paper is to quantify numerically the uncertainty in the attenuation performance of a cantilever beam with two piezoelectric patches and a resonant random "RL"shunt. The nominal values of the inductance and resistance are chosen in order to achieve maximum energy dissipation of the second mode of the cantilever beam.

Key-words. Vibration reduction, Piezoelectric shunting, Sensitivity analysis, Uncertainty quantification.

1 Introduction

Piezoelectric materials are proposed for many applications, especially in the field of dynamics where their properties of coupling mechanical stress and strain with an electric circuit are used to detect, measure, or control the vibrations. Some of today's active research fields that use piezoelectric materials are energy harvesting, passive or semi-passive structural vibration damping, active vibration control, shape adaptation and structural

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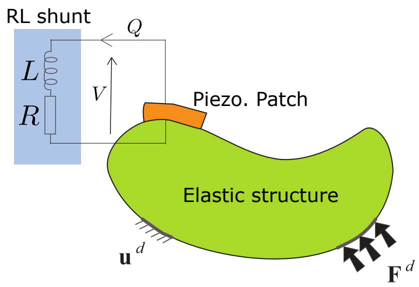
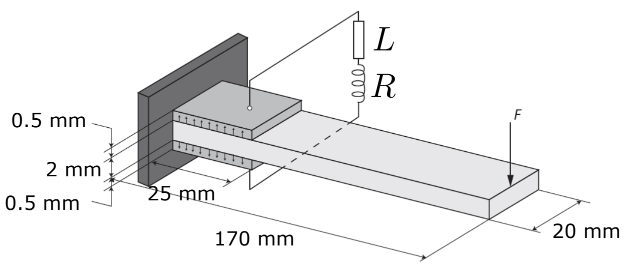
health monitoring [11]. This work deals with passive reduction of structural vibration by means of shunted piezoelectric patches. The problem consists of an elastic structure with surface-mounted piezoelectric patches. It is an electromechanical systems, i.e., a coupled systems in which energy can be transferred between the mechanical and electromagnetic parts [4, 5]. The piezoelectric elements are connected to a resonant shunt circuits ("LR" shunt) in order to damp specific resonant frequencies of the structure. When the shunt is perfectly tuned to the resonance frequency to control, the vibration attenuation is optimal. On the other hand, a slight shift generates a significant loss of the damping performance of the system [1]. The purpose of this work is to evaluate the sensitivity of the system vibration response (and thus the loss of attenuation) to uncertainties in the electrical parameters of the shunt (inductance and resistance), around their optimum value. Since uncertainties are unavoidable, this paper discusses how important they are. To quantify the vibration attenuation, two nondimensional variables are defined relating the amplitude of the frequency responses in without "RL" shunt (short-circuit) and with shunt "RL". Another parameter observed is related with the difference between the maximal frequency response for the short-circuited system and the the maximal frequency response for the shunted system. The objective of the paper is to quantify the uncertainties in these three parameters of interest when the inductance and resistance of the shunt are random. To exemplify numerically the stochastic vibration attenuation, the methodology is applied to a cantilever beam with two piezoelectric patches and a resonant "RL"shunt. The nominal values of the inductance and resistance are chosen in order to achieve maximum energy dissipation of the second mode of the coupled system. The paper is organized as follows. In section 2, the dynamics of the coupled electromechanical system is presented. In section 3, the variables used to quantify the vibration attenuation are defined. The stochastic models to the electrical parameters of the shunt are presented in section 4. The numerical results of the uncertainty quantification of the vibration attenuation are discussed in section 5.

2 Electromechanical Formulation



(a)



(b)

Figure 1: (a) An arbitrary structure with a piezoelectric patch connected to a resonant shunt. (b) Model problem: cantilever beam with a piezoelectric patch connected to a resonant shunt [2].

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We consider an arbitrary elastic structure with one piezoelectric patch, sketched in Fig. 1(a). We denote by $\mathbf{u}(\mathbf{x},t)$ the displacement of a point $\mathbf{x}$ of the structure, at instant t. A resonant shunt is connected to the piezoelectric patch; V denotes the voltage between the electrodes, which is also the shunt terminal voltage, and Q is the electric charge in one of the electrodes. Considering the convention of sign for V on Fig. 1(a), Q is precisely the charge in the upper electrode. Several models for this coupled electromechanical system can be obtained, either in an analytic fashion or using a finite-element discretization [3]. Then, a reduced order model can be obtained by expanding the displacement $\approxeq$ onto N vibration eigenmodes:

$$\mathbf{u}(\mathbf{x},t) = \sum_{1}^{N} \mathbf{\Phi}_i(\mathbf{x}) q_i(t) \tag{1}$$

One can show that the modal coordinates $q_i(t)$ are solutions of a problem of the form:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \chi_i V = F_i \quad \text{for } i \in \{1,\cdots,N\} \tag{2}$$

$$CV - Q + \sum_{1}^{N} \chi_i q_i = 0 \tag{3}$$

The electromechanical model of the problem is thus described by N modal equations, corresponding to the balance law of mechanical forces, and one electrical equation, associated with the balance of electric charges on the piezoelectric electrodes. Here, the short-circuit eigenmodes are used. They are the vibration modes of the structure with its piezoelectric patch short-circuited ($V = 0$). Thus, $(\omega_i, \mathbf{\Phi}_i)$ denotes the angular natural frequency and mode shape of the corresponding ith mode, respectively. The electromechanical coupling appears in those equations by a modal coupling coefficient χ_i, that characterizes the energy transfer between the ith mode shape and the piezoelectric patch. The electric capacitance of the patch is denoted by C. Finally, a modal structural damping term, of factor ξ_i, has been added. It is convenient to rewrite equation Eq. (2) with Q as the electrical unknown. By introducing equation Eq. (3) into Eq. (2) to eliminate V, one obtains the following set of equations, equivalent to equation Eq. (2):

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \frac{\chi_i}{C}\sum_{1}^{N} \chi_i q_i + \frac{\chi_i}{C} Q = F_i \quad \text{for } i \in \{1,\cdots,N\} \tag{4}$$

In the case of a resonant shunt, where the electric circuit connected to the piezoelectric patches is a resistance R and an inductance L in series (1(a)), the relationship between V and Q is $V = -R\dot{Q} - L\ddot{Q}$. Then, the dynamics of the coupled electromechanical system becomes [8].

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i - \frac{\chi_i}{C}\sum_{1}^{N} \chi_i q_i + \frac{\chi_i}{C} Q = F_i \quad \text{for } i \in \{1,\cdots,N\} \tag{5}$$

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C} Q - \sum_{1}^{N} \chi_i q_i = 0 \tag{6}$$

3 Variables used to quantify the vibration attenuation

In this section, formulas derived in the previous section are applied to the analysis of a cantilever beam sketched in Fig. 1(b). The geometry and constitutive parameters of the steel beam and of the piezoelectric patch can be found in [2]. The beam is excited by a sinusoidal force applied at its tip normal to the beam. The resonant shunt is tuned in order to achieve maximum energy dissipation of the second mode (which occurs at frequency ω_2^{SC}). The nominal values considered to the inductance and to the resistance are $L =$ 14.8 H and $R = 8000\Omega$. A reduced order model is constructed with two modes. Figure 2(a) shows the frequency responses, in the short-circuited (h^{SC}) and shunted (h^{LR}) cases. One of the parameters used to quantify the vibration reduction is related with the reduction of the frequency response amplitude for the frequency ω_2^{SC}, i.e., $h^{SC}(\omega_2^{SC}) - h^{LR}(\omega_2^{SC})$, illustrated in Fig. 2(b). Another parameter is related with the difference between the maximal frequency response for the short-circuited system (which occurs at ω_2^{SC}) and the the maximal frequency response for the shunted system (which occurs at ω_2^{LR}). These two parameters of interest and the shift in frequency due to the coupling are defined as nondimensional variables given by:

$$p_1 = \frac{\left|h^{SC}(\omega_2^{SC})\right| - \left|h^{LR}(\omega_2^{SC})\right|}{\left|h^{SC}(\omega_2^{SC})\right|} \qquad p_2 = \frac{\left|h^{SC}(\omega_2^{SC})\right| - \left|h^{LR}(\omega_2^{LR})\right|}{\left|h^{SC}(\omega_2^{SC})\right|} \qquad \Delta\omega = \frac{\omega_2^{SC} - \omega_2^{RL}}{\omega_2^{SC}} \tag{7}$$

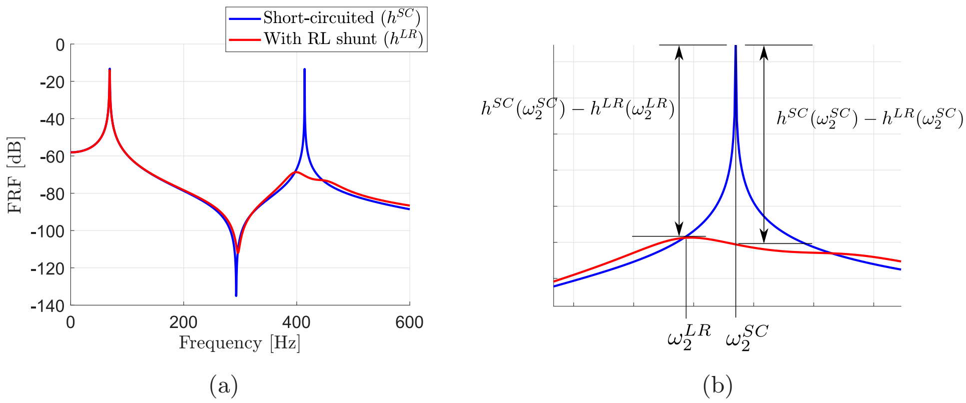

Figure 2: (a) Frequency responses to the short-circuited (h^{SC}) and shunted (h^{LR}) systems. (b) Zoom over the second mode shown in Fig. 2(a).

4 Stochastic model to the electrical parameters of the shunt

To evaluate the sensitivity of the system vibration response to uncertainties in electrical parameters of the shunt around their optimum value, the inductance and the resistance are considered to be random. Other sources of uncertainties exist in this electromechanical system, such as the glue used to attach the PZT patches in the beam (affecting the

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electromechanical coupling). However this work focus on the uncertainties in the inductance and the resistance. They are modeled as uniform random variables, called as $\mathbb{L}$ and $\mathbb{R}$ respectively. It is considered that the inductance has mean $\overline{L} = 14.8$ H and support $[0.9,\ 1.1]\overline{L}$. The resistance has mean $\overline{R} = 8000\Omega$ and support $[0.9,\ 1.1]\overline{R}$. Due to the assumption, the dynamics of the coupled electromechanical system becomes a stochastic dynamics and the parameters used to quantify the vibration reduction become random variables, $\mathbb{P}_1$, $\mathbb{P}_2$ and $\Delta\Omega$.

5 Uncertainty quantification of the vibration attenuation

To estimate statistics and histograms of these parameters, i.e., quantify their uncertainties [6, 7], the frequency response of the shunted stochastic system is evaluated 10^4 times using independent realizations of the inductance and resistance generated with the Monte Carlo method [9, 10]. Figure 3(a) and 3(b) shows the normalized histogram of the inductance and resistance samples. Figure. 4(a) shows the envelope graph of the stochastic frequency response. The normalized histograms of $\mathbb{P}_1$, $\mathbb{P}_2$, $\Delta\Omega$ and $[\mathbb{P}_2\ \Delta\Omega]^T$ are shown in Figs. 4(b), 5(a), 5(b) and 6.

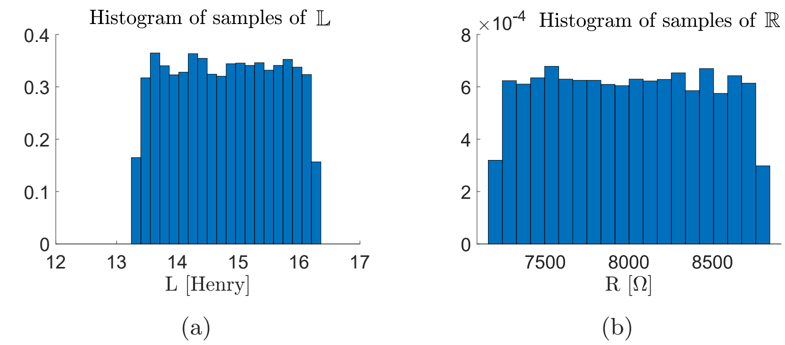


Figure 3: Normalized histogram of the (a) 10^4 inductance samples and (b) 10^4 resistance samples.

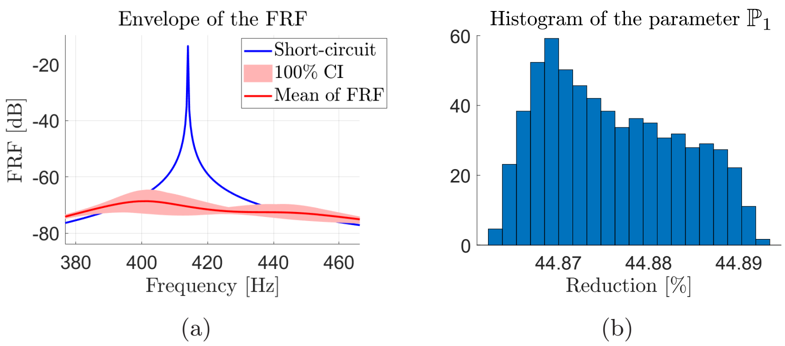
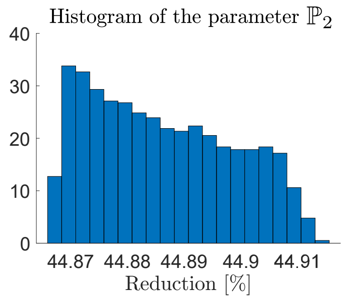
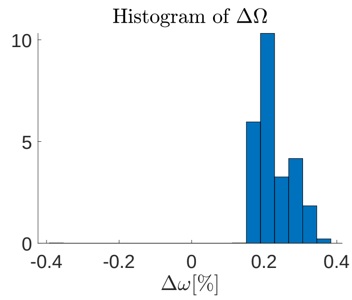


Figure 4: (a) Envelope graph of the stochastic frequency response. (b) Normalized histogram of the 10^4 $\mathbb{P}_1$ samples.



(a)



(b)

Figure 5: (a) Normalized histogram of the (a) 10^4 $\mathbb{P}_2$ samples and (b) 10^4 $\Delta\Omega$ samples.

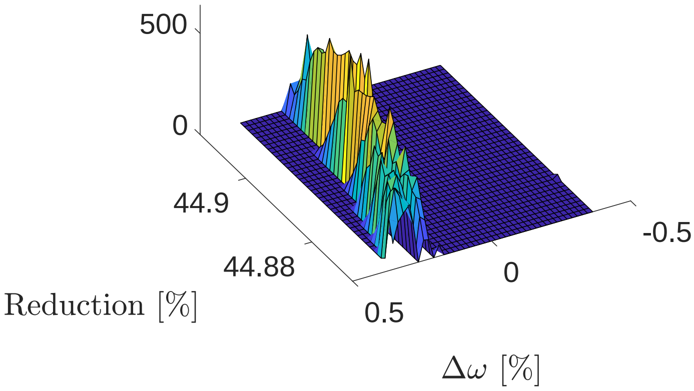


Figure 6: Normalized histogram of the 10^4 samples of the vector $[\mathbb{P}_2\ \Delta\Omega]^T$.

6 Conclusions

In this paper, the stochastic damping performance of a cantilever beam with two piezoelectric patches and a resonant random “RL”shunt is investigated. The nominal values of the inductance and resistance are chosen in order to achieve maximum energy dissipation of the second mode of the cantilever beam. Two nondimensional variables are defined to quantify the vibration attenuation. These the variables relate the amplitude of the frequency responses in without “RL” shunt (short-circuit) and with shunt “RL”. To evaluate the sensitivity of the system vibration attenuation to uncertainties in the “RL”shunt, the inductance and the resistance are considered to be random. Statistics and histograms of the parameters used to quantify the vibration attenuation are computed with Monte Carlo simulations.

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