A verification of the Cosserat rod implementation for drill-string dynamics under large displacement conditions

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Abstract. A cantilever beam modelled as a Cosserat rod, subjected to an external eccentric load is studied as a means to verify the capability of the current implementation to handle large displacement conditions. It is shown that the present strategy based on the Cosserat theory succesfully overcomes the known limitations of other implementations, such as the Modified Cosserat Rod Element (MCRE).

Palavras-chave. Cosserat rods, drill-string, geometric non-linearity

1 Introduction

Oil production has been increasing over the last years [15], and it has become the most important energy source for a forseeable future [7]. Drilling is essential to the exploitation of oil and gas reservoirs, with a great impact on the associated costs to oil production. For this reason, drill-strings have been the subject of numerous studies in order to gain insight into the processes that affect their expected life. Particularly, premature failure of the drill-string as well as the drill-bit, may happen due to undesired vibrations of the drill-column, as well as reduced performance, i.e. rate-of-penetration. As a consequence, companies are interested in new methods and technologies to identify unwanted vibrations and to supress them [4]. For these reasons the development of dynamic models for drill-strings is essential to understand and control vibrations.

The drilling assembly used in the drilling process is composed of the following three main components: a drill-string (DS), in charge of driving the necessary torque to the

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cutting tool; a bottom hole assembly (BHA) that, along with the DS, provide weight to the bit; and the drill-bit in charge of performing the actual cutting of the rock. It is worth to mention that there are other important tools that comprise the drilling assembly, such as measure-while-drilling devices, stabilizers and others, which may play an important role in the dynamics of the drilling device, but will not be studied in detail in the present work.

The dynamics of the drilling assembly has been the subject of numerous investigations over the last decades, and various approaches based on different theories have been employed to perform such analysis. A comprehensive review of the many models available for drill-string dynamics is presented in [4].

Generally speaking, the models found in the literature can be classified in two groups depending on how the bending effects are considered, namely the soft-string and stiff-string models. The first group, includes those approaches that neglect the bending stiffness of the drill-string, such as in [11, 1, 10, 13]. In this group, the fact that the drill-string is a very slender structure is employed to disregard the dynamic in the lateral planes. In contrast, the second group belongs to the stiff-string models, which do not neglect the bending stiffness and, as a consequence, take into account lateral displacements, as in [8, 9].

On the other hand, DS models from any of the aforementioned groups can be either lumped (discrete) or continuous. The choice of the theoretical framework is usually based upon the complexity involved in the resolution of each model, and also on the time spent to perform the required computations.

As already stated, many different theories have been used in the analysis of drill-string dynamics. For example, in [5] a continuous Euler beam theory is employed to deal with a vertical straight borehole geometry. The special theory of Cosserat rods is employed in [14, 12] to analyse the dynamics of a drill-string in a straight vertical borehole, though the method presented should, theoretically, be capable of dealing with arbitrary configurations.

Lastly, [7] studies drill-string dynamics via two different approaches: a Cosserat rod model based on the Modified Cosserat Rod Element (MCRE) finite element method (FEM) implementation, and an elasticity theory via FEM. Then, [7] presents an analysis of the capabilities of each model and concludes that the MCRE approach is not suitable for arbitrary drill-string configurations, as it is prone to instabilities in situations that involve large displacements. For this reason, the authors of [7] chose to employ the 3D elasticity theory implemented with the FEM in Abaqus, a commercial software.

The objective of the present work is to show that the Cosserat rod theory implementation employed in [6] can successfully overcome the limitations found with the MCRE-based model for drill-string dynamics. The theory is implemented in COMSOL [2], another commercial FEM software.

2   The Cosserat rod theory

The formulation for the special Cosserat rod theory is presented. For such purpose, a sketch of a section of a rod is presented along with the applied and distributed forces and moments that will be considered in the formulation.
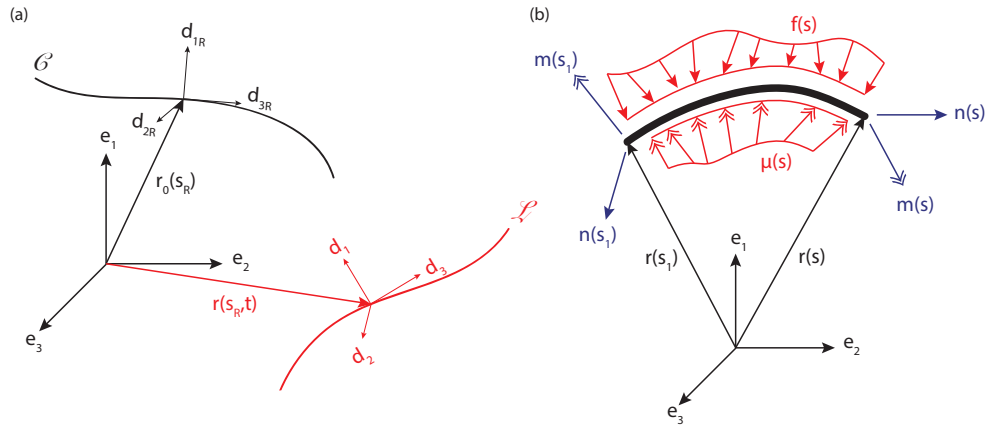
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Figure 1: (a) Curves $\mathscr{C}$ and $\mathscr{L}$ represent the reference and current centreline configurations, respectively; Vectors d_i the current moving frame; d_{iR} the reference moving frame; e_i an arbitrary inertial frame. (b) Equilibrium of a rod segment. Vectors $\mathbf{n}$, $\mathbf{m}$ represent applied loads and moments, respectively; $\boldsymbol{\mu}$, $\mathbf{f}$ are distributed forces and moments, respectively.

The model is solved using [2], a commercial FEM environment. The equations are manually introduced in their weak form, given that no built-in module is included for this theory. For the sake of completeness all the required mathematical expressions are included below. A more detailed definition and derivation of the previous equations is presented in [6].

Let $\mathbf{r}(s_R, t)$ be a vector field that defines the current position of the centreline of a rod, $\mathbf{d}_i\big(q_a(s_R, t), q_b(s_R, t), q_c(s_R, t), q_d(s_R, t)\big)$ the directors of a moving frame that is fixed to the cross-sections, $\mathbf{v}(s_R, t)$ a tangent vector to the centreline, and let $\mathbf{r}_R$, $\mathbf{d}_{iR}$, $\mathbf{v}_R$ be analogous vector fields defined for the reference configuration. Then, the weak formulation reads as follows

$$
\begin{aligned}
&\int\big(-\phi'_{rx}\, n_x + \phi_{rx}\, f_x - \phi_{rx}\, \rho_0\, A\, \ddot{r}_x\big)ds_R &&+&& \int\big(-\phi'_{ry}\, n_y + \phi_{ry}\, f_y - \phi_{ry}\, \rho_0\, A\, \ddot{r}_y\big)ds_R \\
&\int\big(-\phi'_{rz}\, n_z + \phi_{rz}\, f_z - \phi_{rz}\, \rho_0\, A\, \ddot{r}_z\big)ds_R &&+&& \int\big(-\phi'_{qb}\, m_x + \phi_{qb}\,(\eta_x + \mu_x) - \phi_{qb}\, \dot{\kappa}_x\big)ds_R \\
&\int\big(-\phi'_{qc}\, m_y + \phi_{qc}\,(\eta_y + \mu_y) - \phi_{qc}\, \dot{\kappa}_y\big)ds_R &&+&& \int\big(-\phi'_{qd}\, m_z + \phi_{qd}\,(\eta_z + \mu_z) - \phi_{qd}\, \dot{\kappa}_z\big)ds_R \\
&\int\big(\phi_{qa}(q_a^2 + q_b^2 + q_c^2 + q_d^2 - 1)\big)ds_R &&+&& \phi_{rx}\, n_x\Big|_0^L + \phi_{ry}\, n_y\Big|_0^L + \phi_{rz}\, n_z\Big|_0^L \\
&+\phi_{qb}\, m_x\Big|_0^L + \phi_{qc}\, m_y\Big|_0^L + \phi_{qd}\, m_z\Big|_0^L &&=&& 0
\end{aligned}
$$

$$(1)$$

In this equation ρ_0 is the reference mass density, A the cross-section area, $\phi_{(\cdot)}$ are the test functions, $\kappa_{(\cdot)}$, represent the components of the angular momentum, $\eta_{(\cdot)}$ the vector components of the product $\mathbf{r}^{*'} \times \mathbf{n}^*$, and $n_{(\cdot)}$, $m_{(\cdot)}$, $f_{(\cdot)}$, $\mu_{(\cdot)}$ are the components of an applied force, applied moment, distributed force, distributed moment, respectively, with all the components expressed in an inertial frame.

The constitutive relations employed take the form

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$$\mathbf{n} = \tilde{K}(\mathbf{v} - \mathbf{v}_R) + \tilde{K}_t(\dot{\mathbf{v}}) \tag{2}$$

$$\mathbf{m} = \tilde{J}(\mathbf{u} - \mathbf{u}_R) + \tilde{J}_t(\dot{\mathbf{u}}) \tag{3}$$

where

$$\mathbf{u} = \frac{1}{2} \sum_{i=1}^{3} \mathbf{d}_i \times \frac{\partial \mathbf{d}_i}{\partial s_R} \tag{4}$$

$$\mathbf{u}_0 = \frac{1}{2} \sum_{i=1}^{3} \mathbf{d}_{iR} \times \frac{\partial \mathbf{d}_{iR}}{\partial s_R} \tag{5}$$

and if the reference configuration is chosen so that the directors d_{iR} coincide with the directors of the inertial frame, then

$$
\begin{array}{lll}
d_{1x} = q_a^2 + q_b^2 - q_c^2 - q_d^2 & d_{2x} = 2(q_b q_c - q_a q_d) & d_{3x} = 2(q_a q_c + q_b q_d) \\
d_{1y} = 2(q_a q_d + q_b q_c) & d_{2y} = q_a^2 - q_b^2 + q_c^2 - q_d^2 & d_{3y} = 2(q_c q_d - q_a q_b) \\
d_{1z} = 2(q_b q_d - q_a q_c) & d_{2z} = 2(q_a q_b + q_c q_d) & d_{3z} = q_a^2 - q_b^2 - q_c^2 + q_d^2
\end{array}
\tag{6}
$$

$$q_a^2 + q_b^2 + q_c^2 + q_d^2 = 1 \tag{7}$$

3 Verification of the present Cosserat rod model under large displacements

The deflection of a cantilever beam subjected to an eccentric force has been studied in [7] in order to test the capabilities of a model to handle geometric non-linearities. In the present work, this problem is solved to test the capabilities of the Cosserat rod implementation presented in [6]. A sketch of the beam and its geometrical properties is presented in Fig. 2.

The beam has a length $L = 20$ m, a cross section with diameter $d = 0.2$ m, Young elastic modulus $E = 210$ GPa, and an eccentricity $e = L/100$. For this case, the classical Euler buckling theory predicts a maximum critical load $P_{cr} = \pi^2 EI/(2L)^2$. The eccentric force is varied by considering a parameter η, with $F = \eta \cdot P_{cr}$, and $M = \eta \cdot P_{cr} \cdot e$.

An analytical solution to this problem is reported by [7], based on the solution presented in [3]. The expression is obtained by considering second-order effects in Bernoulli-Euler beam theory, and reads as

$$\delta = \frac{M}{F} \left(\sec \left(L \sqrt{\frac{F}{EI}} \right) - 1 \right) \tag{8}$$
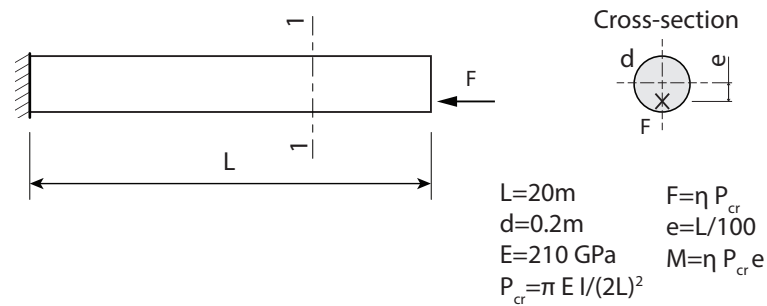
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Figure 2: Sketch of a cantilever beam with length L, diameter d, eccentricity e, Young elastic module E, under the effects of an eccentric compressive force F. M is the equivalent bending moment that results from the application of the eccentric load F.
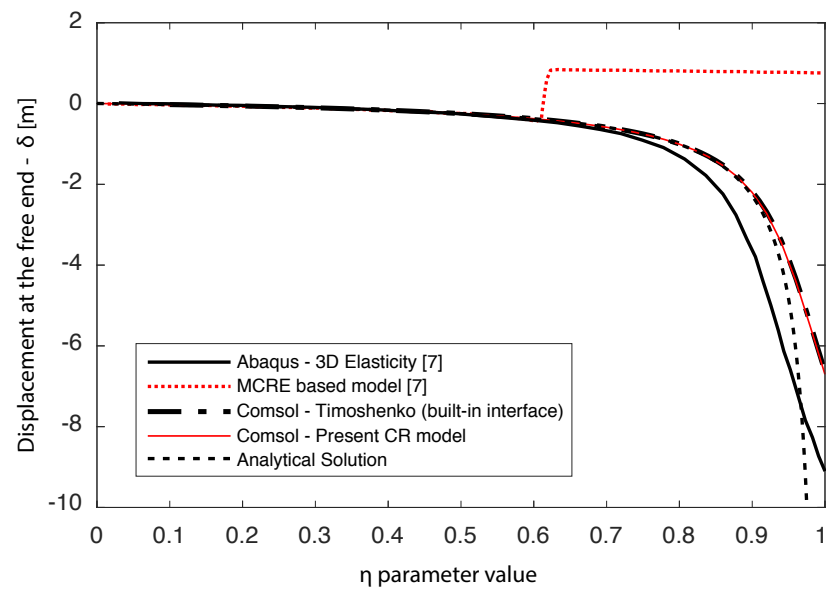
4 Results

Figure 3: Results for the displacement of the cantilever beam at the free end. Solution for the built-in [2] Timoshenko theory that accounts for large displacements (long-dash-short-dash black line); present Cosserat rod implementation (continuous red line); 3D model based on the elasticity theory from [7] (continuous black line); MCRE model from [7] (dotted red line); analytical solution (dashed black line) following (8).

The results are illustrated in Figure 3. The solution for the built-in Timoshenko theory present in [2] considering a co-rotational formulation to account for large displacements is

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shown in long-dash-short-dash black line; in red line the present Cosserat theory implementation is depicted; in continuous black line the results published in [7] for a 3D model based on the elasticity theory are presented; in dotted red the line the solution for the MCRE based model from [7]; finally, in dashed black line the results from the analytical solution following (8).

It is observed that the Cosserat rod implementation applied to the present problem agrees with the solution obtained by means of the built-in COMSOL model. In contrast to the MCRE approach, no singularity is observed with the current CR implementation. Moreover, the results satisfy the analytical solution with no appreciable differences for $\eta \leq 0.9$.

Note that the analytical solution tends to infinity as $\eta \to 1$. This result is due to the hypothesis introduced in the second-order Bernoulli-Euler theory, where the expression for the curvature is simplified for the condition of small rotation angles, thus rendering the solution invalid for large deflections.

5   Conclusion

In this work, the problem of a cantilever beam subjected to an eccentric compressive force is solved as a means to verify the response of the present implementation of the Cosserat rod (CR) model under large displacement conditions. The results agree with those obtained by the built-in module of [2], employing a co-rotational Timoshenko theory with geometric non-linearity. Also, is it observed that the CR model response qualitatively follows the same behaviour as the solution for the 3D elasticity theory reported in [7]. Therefore, the authors find it feasable to employ the current CR implementation to deal with drill-string dynamics, even in situations that require accounting for large deformations (geometrical non-linearities). Moreover, the present CR implementation overcomes the limitations and singularities analysed in [7], for the solution based on the Modified Cosserat Rod Element (MCRE).

6   Acknowledgements

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