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Energy analysis of an electromagnetic loudspeaker

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Abstract. An electromechanical system is composed by two subsystems with distinct origins: one of a mechanical nature and another of electromagnetic nature. The energies in the system have also different origins. Some of them are mechanical, as kinetic and potential, and others are electromagnetic, as magnetic and electrical. For a proper description of an electromechanical system dynamics it is not sufficient to describe each subsystem separately. Coupling terms must be considered in the system dynamics. These terms characterize the mutual influence between the two subsystems and the interplay of the energies of the system. The objective of this paper is to analyze from an energetic point of view an electromechanical system. This paper shows how the dynamics of an electromechanical system can be constructed by the definition of the energies that are present in the system and their interplay using the Lagrangian method. To exemplify, an electromagnetic loudspeaker will be analyzed. Its dynamics will be constructed and numerical integrated in order to make an energetic analysis.

Key words. Lagrangian, energy, co-energy, electromechanical, transducer.

1 Introduction

Electromechanical systems are composed by two subsystems, a mechanical and an electromagnetic. They can be found in several applications used in daily life. However, even though they are so common, it is still a challenge to find references correctly describing their dynamics. Several published papers, books and thesis present serious mistakes in the description of the dynamics of an elestromechanical. A common error found in the literature is the neglection of the dynamics of the electromagnetic subsystem and its interactions in the system dynamics (see reference *Cveticanin, L., Zukovic, M., Balthazar, J. M.* from [5]). The two subsystems that compose an electromechanical system have coupling terms that cause interaction between them. When the electromagnetic subsystem and these coupling terms are neglected, the electromechanical system becomes a purely mechanical system described by mechanical variables. The recent published paper [3] discusses about some of the references with mistakes and shows how the neglection of the electromagnetic and coupling terms changes the dynamics.

The objective here is to make a step by step of how to describe properly the dynamics of an electromechanical system using the Lagrangian method, also seen in [4] and [8]. To accomplish this gol, the dynamic equations of an electromagnetic loudspeaker are going to be deducted and analysed.

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The electromagnetic loudspeaker analyzed in this paper is presented in section 2. The variables, parameters and conditions required to use this system are also given in section 2. The coupling term that produces the interaction between the mechanical subsystem and the electromagnetic subsystem in this loudspeaker is a transducer. This element is presented in section 3. In section 4, the energies that are presented in the system are defined and used in the construction of the system dynamics by the Lagrangian method.

After all the calculations, an energy analysis will be made in section 5 to compare the different types of energy (kinetic, potencial, electric and magnetic) and show theis interplay by the results of numerical integrations of the system dynamics.

2 Electromagnetic loudspeaker

To exemplify the interaction of the two subsystems that compose an electromechanical system, an electromagnetic loudspeaker will be simulated and discussed. This loudspeaker is illustraded in figure 1.

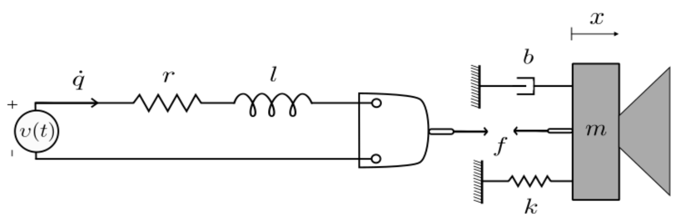

Figure 1: Electromagnetic loudspeaker. [7]

The system is composed by a mechanical part (a mass m, a spring of constant k and a damper of constant b, where the last two are simulating a membrane that dislocates the air), an electromagnetic part (a voltage source v in series with an RL circuit, which means an inductor of inductance l and a resistor of resistance r) and an element called moving-coil transducer (with transducer constant ϱ, explored topic in section 3) that couples the subsystems. Two variables are used to describe the system configuration. One of them is mechanical, it is called x the displacement of the mass m from the mechanical subsystem's equilibrium point, and the other is electromagnetic, the charge q passing through the circuit.

It is important to stand out the fact that the displacement x has nothing to do with the sound produced by the loudspeaker, it is merely the displacement of the mass m from the chosen equilibrium point.

3 Moving-coil transducer

A moving-coil transducer is an energy transformer element of a system that converts electrical power into mechanical power or vice versa and can not store energy. In the loudspeaker case, the current $\dot{q}$ originated by the potencial difference e passing through the ends of the circuit is being converted into a displacement x. The transducer's elements and its symbolic representation are ilustrated in figure 2.

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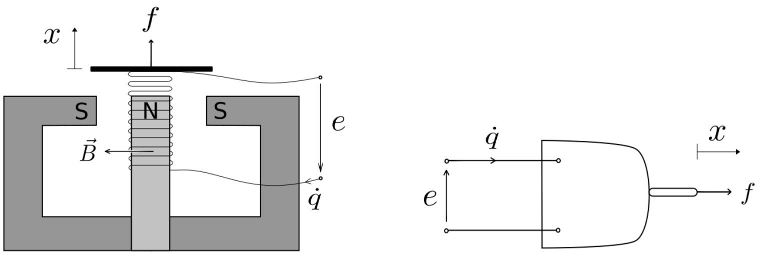

Figure 2: A moving-coil transducer and its symbolic representation, respectively. [7]

A coil is passing around one of the poles from a magnet that generates a magnetic flux density B. Being f the force to keep the coil's equilibrium (it means, opposite to the electromagnetic forces) and knowing that the coil is free to move in the direction of f, it is possible to obtain the magnetic co-energy [1]:

$$U_m^*(x, \dot{q}) \;=\; \varrho \dot{q} x, \tag{1}$$

where ϱ is called the transducer constant and is given by:

$$\varrho \;=\; 2\pi n r B. \tag{2}$$

Here, n is the number of turns of the coil, r is the radius of the coil and, therefore, $2\pi nr$ is the coil's length that is immersed in the magnetic flux B. Also, x_0 in equation 1 depends on the chosen parametrization for the system.

4 Lagrangian formulation for an electromagnetic loudspeaker

The Lagrangian for an electromechanical system [7] is written as:

$$\Gamma \;=\; T^* - V + E_m^* - E_e \pm U^*, \tag{3}$$

where T^* is the kinetic co-energy, V the potential energy, E_m^* the magnetic co-energy and E_e the electric energy.

The coupling term U^* can have an electric or magnetic origin and it's signal depends on this fact. If it is transmited as a magnetic coupling (U_m^*), the signal is positive and if it is transmited as an electric one (U_e^*), the signal is negative, shown in the next equations:

$$\Gamma \;=\; T^* - V + (E_m^* + U_m^*) - E_e, \tag{4}$$

$$\Gamma \;=\; T^* - V + E_m^* - (E_e + U_e^*). \tag{5}$$

Being z_i a generalized coordinate of the system, each differential equation of the system dynamics can be found by:

$$\frac{d}{dt}\left(\frac{\partial \Gamma}{\partial \dot{z}_i}\right) - \frac{\partial \Gamma}{\partial z_i} \;=\; \frac{d\delta W}{d\delta z_i}. \tag{6}$$

In the case of an electromagnetic loudspeaker, the coupling term in the Lagrangian formulation is given by a moving-coil transducer (explained in section 3), an element that contributes with an energy of magnetic origin. Another example of how the interaction between the two subsystems appears is given in [2], [5] and [6], where the coupling term is now given by a DC motor, also an element that contributes with an energy of magnetic origin.

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Next, the equations that describe the system dinamics for the loudspeaker will be constructed using the Lagrangian formulation [7] [9].

For the mechanical subsystem:

$$T^* = \frac{m\dot{x}^2}{2}, \qquad V = \frac{kx^2}{2}. \tag{7}$$

For the electromagnetic subsystem:

$$E_m^* = \frac{l\dot{q}^2}{2}, \qquad U_m^* = \varrho\dot{q}x, \qquad E_e = 0. \tag{8}$$

So, using equation (3), the Lagrangian is given by:

$$\Gamma = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2} + \frac{l\dot{q}^2}{2} + \varrho\dot{q}x. \tag{9}$$

Obtaining the virtual work for the non-conservative elements of the system:

$$\delta_f = \upsilon\,\delta q, \qquad \delta_d = r\,\dot{q}\,\delta q + b\,\dot{x}\,\delta x, \qquad \delta W = \delta_f - \delta_d = \upsilon\,\delta q - r\,\dot{q}\,\delta q - b\,\dot{x}\,\delta x. \tag{10}$$

For x:

$$\frac{\partial\Gamma}{\partial\dot{x}} = m\dot{x} \rightarrow \frac{d}{dt}\left(\frac{\partial\Gamma}{\partial\dot{x}}\right) = m\ddot{x}, \qquad \frac{\partial\Gamma}{\partial x} = -kx + \varrho\dot{q}, \qquad \frac{d\delta W}{d\delta x} = -b\dot{x}. \tag{11}$$

For q:

$$\frac{\partial\Gamma}{\partial\dot{q}} = l\dot{q} + \varrho x \rightarrow \frac{d}{dt}\left(\frac{\partial\Gamma}{\partial\dot{q}}\right) = l\ddot{q} + \varrho\dot{x}, \qquad \frac{\partial\Gamma}{\partial q} = 0, \qquad \frac{d\delta W}{d\delta q} = \upsilon - r\dot{q}. \tag{12}$$

Substituting equations (11) and (12) into equation (6) for each coordinates used in the system, the dynamic equations for the electromagnetic loudspeaker can be found. It is given by the following initial value problem. Given a source voltage $\upsilon(t)$, find (x, q) such that, for all $t > 0$ with initial conditions $x(0) = x_0, q(0) = q_0, \dot{x}(0) = v_0$ and $\dot{q}(0) = i_0$:

$$\begin{cases} m\ddot{x}(t) + b\dot{x}(t) + kx(t) - \varrho\dot{q}(t) = 0, \\ l\ddot{q}(t) + r\dot{q}(t) + \varrho\dot{x}(t) = \upsilon(t). \end{cases} \tag{13}$$

5 Energy analysis

To analyze the interplay between the different types of energy in this system, a rotine was developed using the software *MATLAB* to simulate how the electromagnetic loudspeaker responds during 15 seconds to a situation where the initial conditions are $x(0) = 1, q(0) = 0, \dot{x}(0) = 0$ and $\dot{q}(0) = 0$. To simulate, the initial value problem that gives the system dynamics was integrated by the $4^{th} - 5^{th}$ order Runge-Kutta method with the *ode45 MATLAB* function. The time-step used was 0.002 seconds and the parameters were chosen for a better interpretation of the results and they are given by: $m = 0.15\ kg, b = 0\ Ns/m, k = 0.10\ N/m, \varrho = 0.30\ mT, l = 1.00\ H, r = 0\ \Omega$ and $\upsilon = 0\ V$.

Figure 3 compares the kinetic co-energy T^* with the potencial energy V and the magnetic co-energy E_m^* with the electric energy E_e. It is possible to notice that the potencial energy reaches its maximum and minimum values when the kinetic energy is in its minimum. Something similar, but not equal, occurs this the magnetic co-energy and electric energy: when E_m^* reaches its maximum,

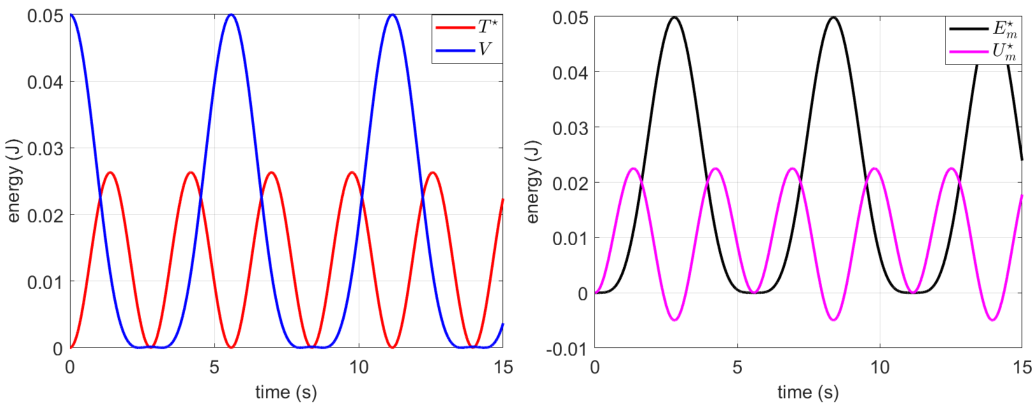

Figure 3: Graphics showing the different types of energies in the system.

E_e is in its minimum (a negative value), but when the magnetic coenergy reaches its minimum, the electric energy is null, reaching a local minimum.

It is also possible to do an energy balance of the system. Using equation (13) and multiplying the first equation by $\dot{x}(t)$ and the second one by $\dot{q}(t)$:

$$\begin{cases} m\ddot{x}(t)\dot{x}(t) + b\dot{x}(t)\dot{x}(t) + kx(t)\dot{x}(t) - \varrho\dot{q}(t)\dot{x}(t) = 0, \\ l\ddot{q}(t)\dot{q}(t) + r\dot{q}(t)\dot{q}(t) + \varrho\dot{x}(t)\dot{q}(t) = \upsilon\dot{q}(t). \end{cases} \tag{14}$$

Adding the two equations found in (14) and making $b = 0, r = 0, \upsilon = 0$ to simplify the analysis:

$$\frac{d}{dt}\left(\frac{m\dot{x}(t)^2}{2} + \frac{kx(t)^2}{2} + \frac{l\dot{q}(t)^2}{2}\right) = 0. \tag{15}$$

It is possible to see in equation (15) that the coupling term of the system dynamics no longer appears. This happens because the moving-coil transducer is an element that does not store energy and, therefore, does not contribute to the energy balance. Figure 4 shows the graphic representation of equation (15), with the sum of the different types of energy: mechanical ($T^* + V$) and electromagnetic (E_m^*). The green line represents this sum and is a constant with a value that depends, in this case, on the parameter k of the system.

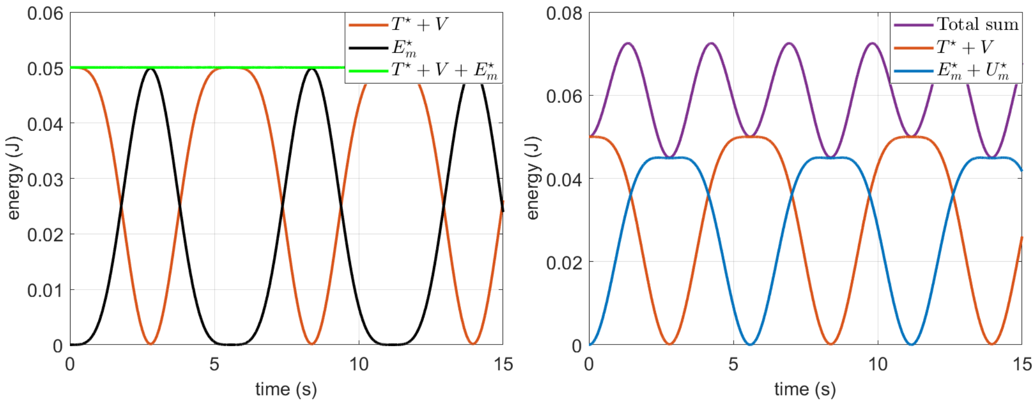

Figure 4: Energy balance and different energies sums, respectively.

Figure 4 also shows two types of sums: one adding the two energies of mechanical origin and another one adding the two energies of electromagnetic origin. It is also shown the total sum of the energies, that is, the sum of all energies regardless of its origin. This last one does not equal a constant, showing once more that U_m^*, energy passing through the transducer, is not stored in this element, it is only transmitted from one subsystem to another.

After this simple example, it is possible to change one of the parameters so a more accurate analysis can be made. Giving a $v = sin(t)$, the same graphics can be analized. The patterns in figures 5 and 6 are repeated every 50 seconds.

Figure 5 shows a different pattern compared to the previous one: now, the potencial energy V reaches its maximums when the kinetic energy T^* is in its minimums and vice versa. The relation between E_m^* and U_m^* also changes: their minimums are always close and the same occurs to the maximums. The fact that U_m^* is, in the most part, negative, shows that this energy is flowing contrary to the one before most of the time.

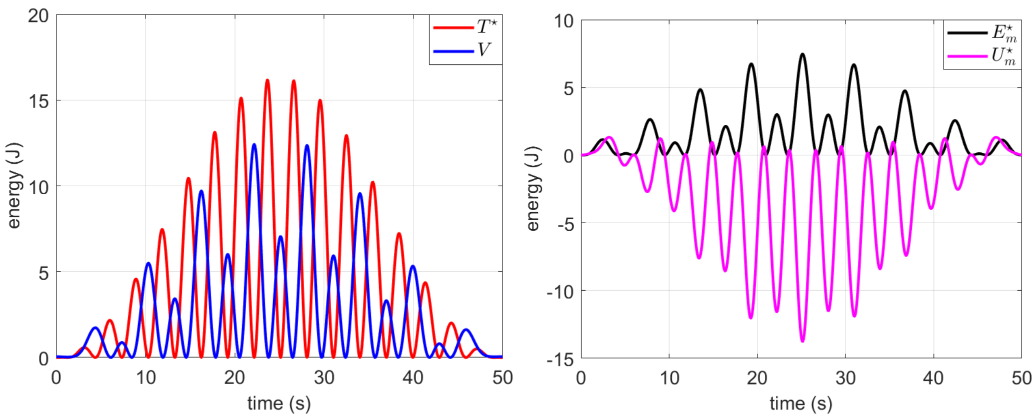

Figure 5: Graphics showing the different types of energies in the system.

The different kinds of energy sums are given in figure 6.

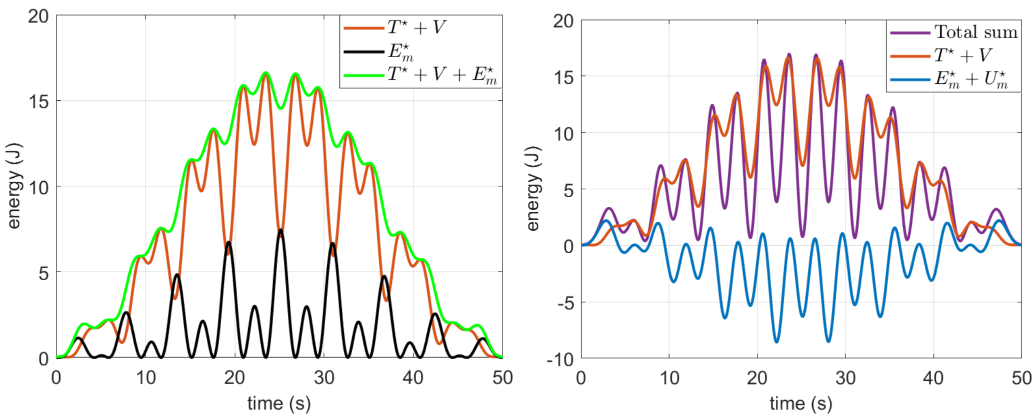

Figure 6: Energy balance and different energies sums, respectively.

6 Conclusions

For a right description of an electromechanical system dynamics, it is important to have in mind the parametrization and the coupling element. This paper showed the correct way of using the Lagrangian method to do that wile using an example of an electromagnetic loudspeaker.

Please remark that it is not sufficient to describe each subsystems (mechanical and electromagnetic) separately, there must be a coulping term between them. This term can have a magnetic origin (as shown with the transducer in the loudspeaker and in [5], [6] and [7]) or an electric origin [7]. It is also explored the fact that a coupling element does not have to store energy, it can only transform it and, therefore, the energy transition in this case is a little bit different from a pure mechanical system.

Acknowledgments

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