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Analysis of the particle-wall interaction in a granular layer of repelling magnetic particles

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In [1], the compression-expansion dynamics of a granular bed composed of cylindrical repelling magnets of diameter D and magnetic moment intensity μ contained in a two-dimensional Hele-Shaw cell is studied. This is a toy model for the compression of a granular bed, since we have only repulsive and friction forces, but no particle-particle contact forces. Nevertheless, this toy model is of interest in itself, because its study can be used for the development of magnetic dampers, for example.

In the aforementioned paper, the authors verified experimentally that the particle-wall interaction force, F_{pw}, is proportional to $\ell^{-3.5}$, where ℓ is the perpendicular distance of the particle to the wall. With the mathematical model considered in Section 4.2 of [1] and considering, instead, a *finite* wall of length L and magnetic moment per unit length M, we have that the point dipole feels a magnetic repulsion force element given by

$$dF_x = -\frac{3\mu_0 M\mu}{4\pi\ell^3}\sin^3\theta\, d\theta, \tag{1}$$

where μ_0 is the magnetic permeability of vacuum and θ is the angle between the wall and the segment connecting the element of integration to the particle. Here, we consider that the magnetization of the wall is given by $M = k\mu/D$, i.e., it is k times stronger than the magnetization μ/D of the dipole. Integrating dF_x with respect to θ, we obtain

$$F_x = -\frac{k\mathcal{J}}{D\ell^3}\left(\frac{\cos^3\theta}{3} - \cos\theta\right)\Bigg|_{\theta_i}^{\theta_f} = -\frac{k\mathcal{J}}{D\ell^3}\left[\frac{1}{3}\cos^3(\theta_f) - \cos(\theta_f) - \frac{1}{3}\cos^3(\theta_i) + \cos(\theta_i)\right], \tag{2}$$

with $\mathcal{J} = 3\mu_0\mu^2/4\pi$ and

$$\theta_i = \arctan\left(\frac{\ell}{fL}\right) = \arctan A, \tag{3}$$

$$\theta_f = \pi - \arctan\left(\frac{\ell}{(1-f)L}\right) = \pi - \arctan B, \tag{4}$$

considering, for simplicity, $\arctan(+\infty) = \pi/2$, f to be the vertical distance of the dipole from the bottom of the bar in terms of the fraction of L that this distance represents and $A, B \in \mathbb{R}_+$. We can then simplify equation (2) using

$$\cos(\arctan x) = \frac{1}{\sqrt{x^2+1}}, x \in (-\pi/2, \pi/2), \tag{5}$$

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and equations (3) and (4) to obtain:

$$F_x = -\frac{k\mathcal{J}}{D\ell^3}\left[\frac{1}{\sqrt{A^2+1}}\left(\frac{3A^2+2}{3A^2+3}\right) + \frac{1}{\sqrt{B^2+1}}\left(\frac{3B^2+2}{3B^2+3}\right)\right]. \tag{6}$$

We plotted F_x *vs.* ℓ in a log-log plot for fixed $L = 100D$ and several values of f, ignoring the constant factor $k\mathcal{J}/D$. We also plotted the experimental dependence obtained in [1], still ignoring the corresponding constant $k\mathcal{J}/D$. The resulting plots are shown in Figure 1.

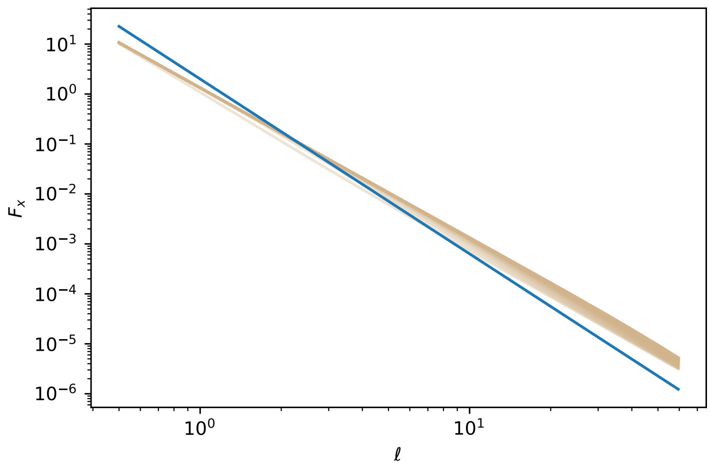

Figure 1: Log-log plot of the repulsion force versus ℓ. In blue we see the experimental fit expected.

Figure 1 shows that $F_x \propto \ell^\alpha, \alpha < 0$, and also that α does not vary much for different values of f, as observed in the tan sheaf of lines; these lines were obtained for f varying from 0.005 to 0.995 with a step of 0.01 and ℓ in the range of 0.5 to 59 with a step of 0.01 as well, with a fit of the form $y = ax^b$. Moreover, we remark that the average of the values of α is ≈ -3.005. This shows that, although the wall is finite, we should not be too far out in modelling it as infinite, since for the infinite wall we have $F_x \propto \ell^{-3}$, with the same constant of proportionality.

In particular, we see that the expected proportionality, $F_x \propto \ell^{-3.5}$, is not recovered. This indicates that the mathematical model proposed in [1] for the particle-wall interaction is not capturing correctly the expected dependence on ℓ. Currently, we are trying to model the walls as finite magnetic bars and use the analytical expressions for the magnetic field to determine the particle-wall interaction. This approach also allows us to take into account field variations near the edges.

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References

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