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Sensitivity of aggregation patterns in magnetic granular assemblies to random perturbations of initial conditions

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In this work, we study how the final steady state configurations of systems composed of three spherical particles that interact mechanically, through contact, and magnetically, via their moments of magnetic dipole, is influenced by random perturbations of the initial geometrical arrangement of particles.

The motion of the particles is limited to a horizontal plane such that gravity does not affect their motion, and their rotation happens solely on the direction normal to the surface of the plane. Each particle i is identified with its magnetic dipole moment $\boldsymbol{\mu}_i$, given by [1]:

$$\boldsymbol{\mu}_i = \mu(\cos(\theta_i), \sin(\theta_i)), \tag{1}$$

where θ_i is the angle of the moment of dipole of particle i with respect to the coordinate system on the plane and μ is the intensity of the magnetic dipoles of the particles. Because of their magnetic moment, the particles will be subjected to magnetic forces and torques [1, 2]. Furthermore, when the particles become in close contact, the mechanical forces and torques acting on the magnetic particles are taken into account by a Discrete Element Method (DEM) algorithm [1, 3]. Further details about the mathematical model describing the interaction of the partcles is presented in detail in [1].

We consider three identical magnetic particles at rest at $t = 0$ placed in an equilateral triangle of size $L_{ini} = 5$, as presented in Figure 1(left). At each vertex of the triangle, we place a particle with a given initial dipole orientation θ_i. As described in detail in our recent work [1], once the particles are allowed to move under the influence of their magnetic interactions, they will evolve to four possible configurations: full dispersion (three particles separated from one another), chains of 2 or 3 particles and rings (three particles forming a closed loop). The final outcome of the system is very dependant on the initial conditions of the system, as the bifurcation diagram in Figure 1(right) reveals.

In order to assess the effect of small perturbations of the initial geometric configuration on the outcome of the system, we will introduce a perturbation on the initial position of the particles given by:

$$\mathbf{S}_i = \mathbf{S}'_i + \varepsilon\hat{\mathbf{p}}_i, \tag{2}$$

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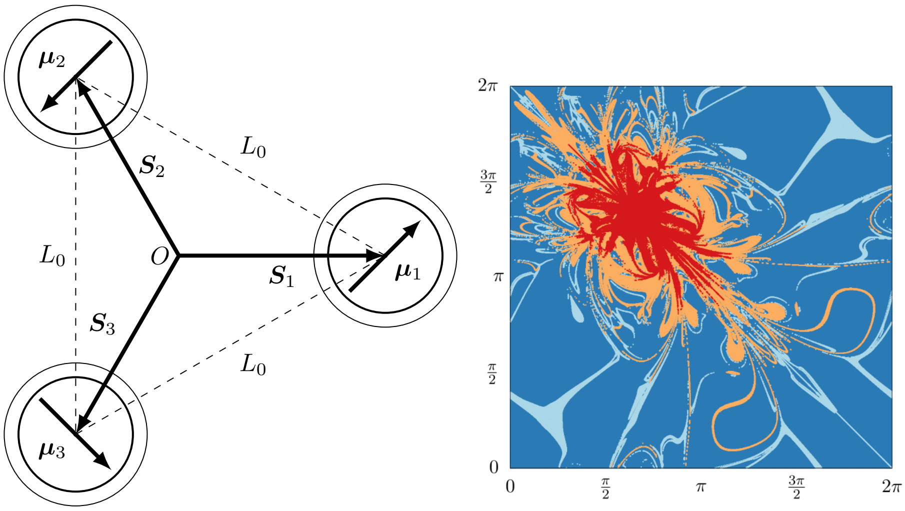

Figure 1: Left: The initial geometrical configuration of the three magnetic particles. Right: Bifurcation diagram for the three particle system obtained for $\theta_1 = 0$. The horizontal axis stands for θ_2 and the vertical axis for θ_3. The color scheme used was dark blue for 3-particle chains, light blue for 3 particles in a ring, yellow for 2-particle chains, and finally red for three dispersed particles. Result extracted from [1].

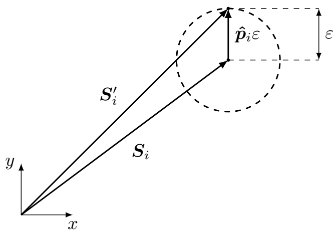

Figure 2: The perturbation of the center of particle i via Eq.(2).

with $\mathbf{S}_i$ denoting the original position of particle i on the vertex of the equilateral triangle, ε the amplitude of the perturbation and $\hat{\mathbf{p}}_i$ denoting the 2D unit random perturbation vector. This would generate a random displacement of size ε of each of the particles i, as shown in Figure 2. We will determine how the bifurcation diagram in Figure 1(right) changes and how the frequency of each of the possible outcomes is influenced by ε.

References

[1] Modesto, J.A.C., Cunha, F.R., Sobral, Y.D., Aggregation Patterns in Systems Composed of Few Magnetic Particles, Journal of Magnetism and Magnetic Materials, 2020. https://doi.org/10.1016/j.jmmm.2020.166664

[2] Rosensweig, R.E., Ferrohydrodynamics, Dover Publications Inc., New York, EUA, 1997.

[3] Schäfer, J., Dippel, S., Wolf, D.E., Force schemes in simulations of granular materials, Journal de Physique I, 6, pp5–20, 1996. https://doi.org/10.1051/jp1:1996129