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Passaging Index - Random Walks applied to complex networks

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Random walks are a well-known process in network science literature [1–3]. They are mathematical objects similar to the Brownian motion concept, a stochastic random process applied in interdisciplinary areas. It is interesting as a diffusion process, modeling the movement of information inside the network with a particle that can walk in any direction, independently from its previous movement.

Random Walks are stochastic processes; given a graph and a starting node, we select a neighbor of it at random and move to this neighbor; then, we select a neighbor of this node at random and move to it, repeating until we reach a certain amount of steps [1].

This work defines an index based on random walks that tells us how many times a walk goes through each node. It is a normalized measure of frequency, which we call *Passaging Index*. Mathematically, it is defined as

$$PI_k = \frac{\text{number of passages}}{\text{total number of steps}}, \tag{1}$$

where PI_k is the *Passaging Index* associated with node k, the number of passages is how many times the node k shows up in the walk, and the total number of steps is the walk's length. It is important to note that the entire sum of the *Passaging Index* has to be one so,

$$\sum_{k}^{N} PI_k = 1, \tag{2}$$

where k is each node, PI is the *Passaging Index*, and N is the set of nodes.

We explore the Random Network model using this method, generated with $N = 100$ nodes and $L = 1547$ edges. After that, we plot in scatter plots comparing the *Passaging Index* with other complex network's indexes: *degree*, *betweenness* and *mean shortest path length*.

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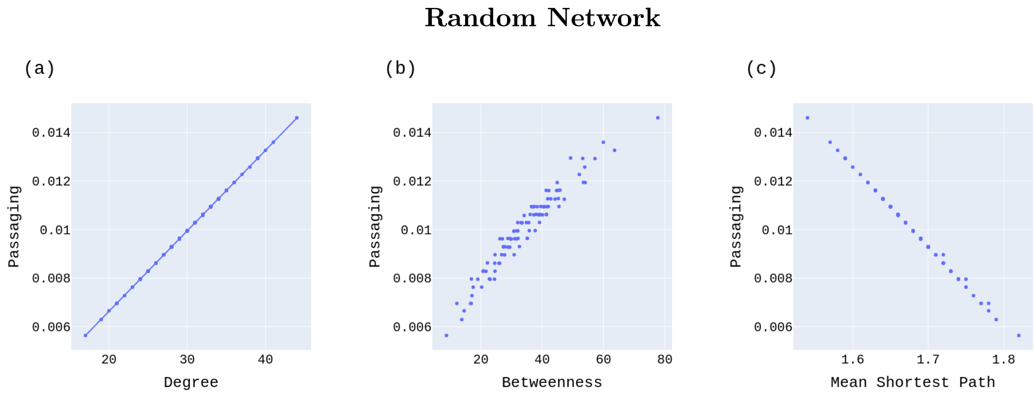

Figure 1: Graph characteristics: N = 100, L = 1475 Density = 0.3, N = 100, L = 1547, $< c > =$ 0.3, $< k > =$ 29.50, D = 2, $< l > =$ 1.68

In Figure 1 we observe a linear relation of this metric with others. Where in image (a) there is a strong relationship, achieving a $R^2 = 1$. As expected, the *Passaging Index* increases with *degree*, *betweenness*, while decreasing with the *mean shortest path length*. This behavior was expected since nodes with more connections attract the walker; however, such strong linearity is intriguing, leading us to think about the stochastic view of already famous metrics.

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