Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 9, n. 1, 2022.

Trabalho apresentado no XLI CNMAC, Unicamp - Campinas - SP, 2022.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics
Preprint

The Maki-Thompson rumor model on a k-regular ring

Ana C. Diaz Bacca,[1]Pablo M. Rodriguez[2]
Universidade Federal de Pernambuco, Recife, PE
Catalina M. Rúa Alvarez[3]
Universidad de Nariño, Pasto, Colombia

We consider the Maki-Thompson rumor model on a k-regular ring, see Figure 1. The basic version of the model is defined by assuming that a population represented by a graph is subdivided into three classes of individuals: ignorants, spreaders and stiflers. A spreader tells the rumor to any of its ignorant neighbors at rate one. At the same rate, a spreader becomes a stifler after a contact with other spreaders, or stiflers. This type of model appeared to describe in a simple way rumor spreading in a population and it was proposed as an alternative to the well-known SIR epidemic model.

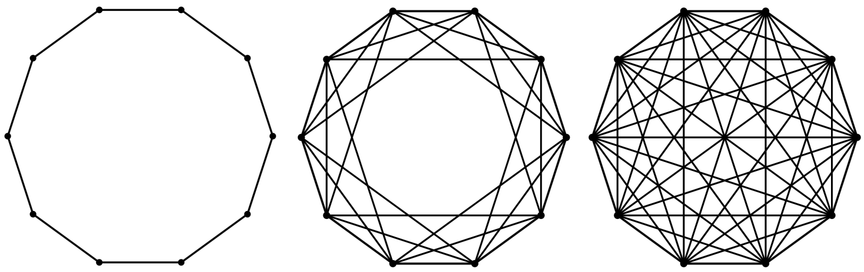

Figure 1: A k-regular ring with $n = 10$ vertices. From left to right we have $k = 2$, $k = 6$ and $k = 9$, respectively.

The first references can be found in [2, 3, 6]. The Maki-Thompson rumor model on a graph G may be defined formally as a continuous-time Markov process $(\eta_t)_{t\geq 0}$ with states space $\mathcal{S} = \{0, 1, 2\}^V$, where V is the set of vertices of G; i.e. at time t the state of the process is some function $\eta_t : V \longrightarrow \{0, 1, 2\}$. We assume that each vertex $v \in V$ represents an individual, which is said to be an ignorant if $\eta(v) = 0$, a spreader if $\eta(v) = 1$, and a stifler if $\eta(v) = 2$. Then, if the system is in configuration $\eta \in \mathcal{S}$, the state of v changes according to the following transition rates

$$\begin{array}{ccccc} & \text{transition} & & & \text{rate} \\ 0 & \rightarrow & 1, & & n_1(v,\eta), \\ 1 & \rightarrow & 2, & & n_1(v,\eta) + n_2(v,\eta), \end{array} \tag{1}$$

where $n_i(v, \eta) = \sum_{u\sim v} 1\{\eta(u) = i\}$ is the number of neighbors of v in state i for the configuration η, for $i \in \{1, 2\}$. Formally, (1) means that if the vertex v is in state, say, 0 at time t then the probability that it will be in state 1 at time $t + h$, for h small, is $n_1(v, \eta)h + o(h)$, where $o(h)$

[1]ana.diaz@ufpe.br
[2]pablo@de.ufpe.br
[3]catalina.rua@udenar.edu.co

represents a function such that $\lim_{h\to 0} o(h)/h = 0$. For a review of recent results for this model on different graphs, we refer the reader to [1, 4, 5, 7].

The main results existing in literature are related to the asymptotic behavior of the final proportion of ignorants, in the case of finite graphs, or the spreading or not of the process, in the case of graphs with infinitely many vertices. In this work we consider G as a k-regular ring with n vertices, and we verify that the model exhibits different behaviors according to k goes from 2 (a ring) to $n-1$ (a complete graph) as we see in Table 1. In the former one can prove that the final proportion of ignorants goes to zero as n goes to ∞. But for the complete graph, it is well-known that such a proportion reaches approximately 20%.

k	2	30	62	122	142	272	422
x_∞	0.9955	0.6224	0.2166	0.2140	0.2122	0.2086	0.2038

Table 1: Simulation results at different values of k.

We discuss the role of k in the asymptotic behavior of the remaining proportion of ignorants. We apply some probabilistic methods to understand the behavior for small and large values of k, and we use computational simulations to support our findings.

Acknowledgements

The authors thank FACEPE for financial support.

References

[1] Elena Agliari et al. “Phase transition for the Maki–Thompson rumour model on a small-world network”. In: **Journal of Statistical Physics** 169.4 (2017), pp. 846–875. DOI: 10.1007/s10955-017-1892-x.

[2] Daryl J Daley and David G Kendall. “Epidemics and rumours”. In: **Nature** 204.4963 (1964), pp. 1118–1118. DOI: 10.1038/2041118a0.

[3] Daryl J Daley and David G Kendall. “Stochastic rumours”. In: **IMA Journal of Applied Mathematics** 1.1 (1965), pp. 42–55. DOI: 10.1093/imamat/1.1.42.

[4] Valdivino V Junior, Pablo M Rodriguez, and Adalto Speroto. “Stochastic rumors on random trees”. In: **Journal of Statistical Mechanics: Theory and Experiment** 2021.12 (2021), p. 123403. DOI: 10.1088/1742-5468/ac3b45.

[5] Valdivino V Junior, Pablo M Rodriguez, and Adalto Speroto. “The Maki-Thompson rumor model on infinite Cayley trees”. In: **Journal of Statistical Physics** 181.4 (2020), pp. 1204–1217. DOI: 10.1007/s10955-020-02623-y.

[6] Daniel P Maki and Maynard Thompson. **Mathematical models and applications: with emphasis on the social life, and management sciences**. Tech. rep. 1973.

[7] Alejandra Rada et al. “The Role of Multiple Repetitions on the Size of a Rumor”. In: **SIAM Journal on Applied Dynamical Systems** 20.3 (2021), pp. 1209–1231. DOI: 10.1137/20M1345657.