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On a model for the fake-news diffusion between two interacting populations

Fabiana Travessini De Cezaro[1] Luverci do Nascimento Ferreira[2] Adriano De Cezaro[3]
IMEF FURG, Rio Grande, RS

Abstract. The dynamics of information propagating among populations that interact might have an enormous impact on public opinion, particularly when such information is false, known as fake news. In this contribution, we propose and analyze the fake news dissemination that occurs when two distinct sub-populations (not necessarily homogeneous) share information, using a reinterpretation of a compartmental model for disease dissemination. We show the model's well-posedness and present numerically simulated scenarios for the dynamics of fake news spreading among populations, with the model parameters associated with some human development indices. The obtained results show that the velocity of the fake news diffusion among the populations is largely impacted by the gap between the human development indices of each population. They also show that a small percentage of control over the information shared by the population leads to a large decrease in the amount and velocity of fake news diffusion.

Key-Words. Fake News, Multi-Population, Model, Dynamics

1 Introduction

The spreading of invented or misleading information, nowadays known as fake news, has always been the subject of human behavior. It gains another dimension with the advent of social media. Such information is produced and disseminated, normally, with the proposal of getting some sort of advantage, for example, to profit from the online number of visitors to the published article or to discredit a person (normally a political opponent in an election) in order to conduct a public opinion poll. It was shown in [9] that fake news normally circulates more broadly, faster, and with 70% more chances of being shared than the truth.

Given the quantity and weight of fake news circulating nowadays, the importance of having information on its dynamics and the mechanisms for limiting its advances and effects in our society becomes evident. It makes the truthfulness of the information that we receive online one of our biggest concerns. Modeling the capability of detecting and the dynamics of spreading false information has received attention recently in [2–4, 6] and references therein.

Since fake news spreading acts like an epidemic, the authors in [2, 4] propose a modification of SIR-type compartmental models for diseases spreading [1] for modeling the diffusion of fake news. Using a linearization around the initial conditions and parameters representing the social development, [2, 4] associated the stiffness ratio of the SIR dynamics with the velocity of the fake news spreading in a population. They show numerically that the larger the stiffness ratio, more quickly the truth is restored.

The main innovation of this paper is to examine the implications for the dynamics of fake news dissemination when two distinct sub-populations share information with each other. In Section 2,

[1]fabi.travessini@gmail.com
[2]luverci@gmail.com
[3]decezaromtm@gmail.com

we describe the proposed model as well as its well-posedness. In Section 3, we analyze some of the capabilities of the proposed model with parameters reflecting some human development indices. The simulated scenarios show that information sharing among populations can have a large impact on the dynamics of the spread of fake news. In particular, it is shown that a small amount of control in the model leads to a large decrease in the amount and velocity of fake news diffusion. In addition, the heterogeneity of the human development index across populations affects the velocity of the spread of fake news, but the inverse of the stiffness ratio is not monotonic, which affects the speed of the spread of fake news, contrary to the conclusions in [2, 4]. In Section 4, we summarize the conclusions and address future directions.

2 The proposed dynamics for fake-news spreading among two interacting populations

In this contribution, we use an alternative interpretation and a generalization of the compartmental SIR model dynamics [1], to describe the diffusion of fake news in a scenario composed of two distinct sub-populations of individuals (let's say P_1 and P_2) that interact. Such reinterpretation consists in assuming that each sub-population P_i, is proportionally subdivided into compartments of individuals $S_i(t)$ that are susceptible to believing in fake news shared by individuals in the $C_i(t)$ compartment who are already convinced that the fake news is true and shear this information with other individuals in the population at some rates β_{ij}, and individuals that could reestablish the truth of the information at a rate of γ_i, denoted by $R_i(t)$, after having been in the $C_i(t)$ compartment, for $i = 1, 2$ at any time $t \geq 0$. We also will analyze the possibility of a pulse control strategy to restore the truth in the sub-population j, for $j = 1, 2$, respectively. Such strategies are implemented or not in a population according to the choice of the parameter $\xi_j \in \{0, 1\}$. Therefore, if $\xi_j = 1$ then there exists (in the case of $\xi_j = 0$ there is no such control) a mechanism acting to reestablish the truth, with the efficacy of ρ_j, in the sub-population j. Using the mass and action laws [1], it is possible to argue that the dynamics shall follow the coupled system of differential equations

$$\begin{aligned}
\dot{S}_j(t) &= S_j(t)(-\beta_{jj}C_j(t) - \beta_{ij}C_i(t)) - \xi_j\rho_j S_j(t) \\
\dot{C}_j(t) &= S_j(t)(\beta_{jj}C_j(t) + \beta_{ij}C_i(t)) - \gamma_j C_j(t) \\
\dot{R}_j(t) &= \gamma_j C_j(t) + \xi_j\rho_j S_j(t)
\end{aligned} \tag{1}$$

and initial conditions

$$S_j(0) = P_j - C_j(0) \geq 0, C_j(0) \geq 0, R_j(0) \geq 0\,, \tag{2}$$

for $i, j = 1, 2$ and $j \neq i$. All the parameters in model (1) are assumed to be constant.

Here we follow the interpretation of the parameters given in [3, 4] and assume that β_{ij} and γ_i are associated with the economic index of development of a population, the *internet penetration index* IPI_i and the *human development index* HDI_i, of each population $i = 1, 2$. In particular, $\beta_{ij} = \sigma_{ij}IPI_j$ and $\gamma_i = \alpha_i HDI_i$, for $i, j = 1, 2$, where the proportions are $\sigma_{ij}, \alpha_i \in]0, 1]$. In general, $\sigma_{ij} > \alpha_i$ since it is easier to spread a lie than reaffirm the truth [9].

2.1 Well-posedness

In this subsection, for sake of completeness, we discuss the well-posedness of a solution for the model (1) with initial conditions (2) and some of its properties that will be used in the forthcoming analysis of fake news spreading.

Lemma 2.1. *Let $P(t) = P_1(t) + P_2(t)$, where $P_j(t)$ is the total of individuals in sub-population $j = 1, 2$. Then $P(t)$ is constant for any $t \geq 0$.*

Proof. Summing up the two sides of (1), we arrive at the conclusion that $\dot{P}(t) = 0$, from where the assertion follows. □

Lemma 2.2. *If a solution $U(t) = [S_1(t), C_1(t), R_1(t), S_2(t), C_2(t), R_2(t)]^T$ of (1) with initial conditions (2) exists, then it is uniformly bounded by $P(0)$. In particular, all the coordinates of $U(t)$ are uniformly bounded.*

Proof. Let $\|\cdot\|_1$ be the $1-norm$ in $\mathbb{R}^n$. It follows that $\|U(t)\|_1 \leq \|P(t)\|_1$ for any $t \geq 0$. Since $P(t)$ is constant (see Lemma 2.1), the assertion follows. □

Proposition 2.1. *Let the map $F(t, U(t))$ define the vector field of the right hand side of the model (1). Then:*

i) $F(t, U(t))$ is continuous at any $t \geq 0$.

ii) There exist constants w_1 and w_2 such that $\|F(t, U(t))\| \leq w_1 + w_2\|U(t)\|$.

iii) $F(t, \cdot)$ is Lipschitz continuous whit respect to the second coordinate.

Proof. The assertion in Item i) follows directly from the fact that each coordinate of $F(t, U(t))$ is given by the sum and product of continuous functions and constant parameters.

The item ii) follows immediately from the definition of $F(t, U(t))$ and the boundedness of each coordinate of $U(t)$ given by Lemma 2.1.

A direct calculation shows that the Jacobian matrix of the system (1) is given by

$$JF(t, U(t)) = \begin{bmatrix} a_{11} & -\beta_{11}S_1(t) & 0 & 0 & -\beta_{21}S_2(t) & 0 \\ -a_{11} - \xi_1\rho_1 & \beta_{11}S_1(t) - \gamma_1 & 0 & 0 & \beta_{21}S_2(t) & 0 \\ \xi_1\rho_1 & \gamma_1 & 0 & 0 & 0 & 0 \\ 0 & -\beta_{12}S_1(t) & 0 & a_{44} & -\beta_{22}S_2(t) & 0 \\ 0 & \beta_{12}S_2(t) & 0 & -a_{44} - \xi_2\rho_2 & \beta_{22}S_2(t) - \gamma_2 & 0 \\ 0 & 0 & 0 & \xi_2\rho_2 & \gamma_2 & 0 \end{bmatrix}$$

where $a_{11} = (-\beta_{11}C_1(t) - \beta_{21}C_2(t)) - \xi_1\rho_1$ and $a_{44} = (-\beta_{22}C_2(t) - \beta_{12}C_1(t)) - \xi_2\rho_2$. Hence, it follows from Lemma 2.1 that there exists a constant $L > 0$ such that $|||JF(t, U(t))|||$ uniformly in $t \geq 0$. Hence, the mean value theorem implies that

$$|||F(t, U(t)) - F(t, \tilde{U}(t))\| \leq L\|U(t) - \tilde{U}(t))\|, \tag{3}$$

concluding assertion iii). □

The theorem that follows is one of the main theoretical results of this contribution.

Theorem 2.1. *Let the general assumptions regarding the model (1) satisfied, with initial conditions (2). Then, there exists a unique continuous and non-negative solution $U(t; \xi_j) := U(t) = (S_1(t), C_1(t), R_1(t), S_2(t), C_2(t), R_2(t))^T \in \mathbb{R}^6$ for any $t \geq 0$, for any choices $\xi_j \in \{0, 1\}$. The solution $U(t; \xi_j)$ depends continuously on the model parameters and initial conditions.*

Proof. Given the results in Proposition 2.1, the theorem statements follow from the standard result of well-posedness for systems of initial value problems, e.g., [8]. □

3 Numerical simulation scenarios for the fake news diffusion

In this section, we presented some numerically simulated scenarios of the model (1), where, for simplicity, the total populations are such that $P_1 = P_2 = 1$. The IVP (1)-(2) are solved using the forward Euler method with step-size $h = 10^{-1}$, in the interval of time $[0, 1000]$, implemented in Python 3.9.

The simulated scenarios presented in this contribution take the IPI_j and the HDI_j, of each population P_j, for $j = 1, 2$, obtained in the annual report of the United Nations Development Programme for the year of 2019, obtained in [4, Table 1], with P_j representing distinct countries, indeed France P_j^1, Brazil P_j^2, India P_j^3 and Mozambique P_j^4 characterizing a distinct variety of socially developing populations. The corresponding values of the parameters in the model (1) are specified in Table 1. Moreover, in all simulated scenarios, we consider that $\beta_{12} = \beta_{11}/10$ and $\beta_{21} = \beta_{22}/10$.

Table 1: Values of the parameters for the population $j = 1, 2$ in the simulated scenarios.

Population P_j^k	β_{jj}	γ_j
P_j^1	0.089	0.009
P_j^2	0.072	0.008
P_j^3	0.035	0.006
P_j^4	0.021	0.005

In the forthcoming analysis, an important quantity is the "time-dependent" inverse of the stiffness ratio of the Jacobian matrix $JF(t, U(t))$ of the model (1), as derived in the proof of Proposition 2.1, given by

$$\tau(t, U(t)) := \frac{|\lambda_{min}(t, U(t))|}{|\lambda_{max}(t, U(t))|} \tag{4}$$

where $\lambda_{max}(t, U(t))$ and $\lambda_{min}(t, U(t))$ are, respectively, the maximum and the minimum non-zero eigenvalues of the Jacobian matrix $JF(t, U(t))$, for any $t \in [0, 1000]$.

Simulated Scenario 1: We present two simulated scenarios in which we have the initial conditions (2) with $C_1(0) = 0.1$ and $C_2(0) = 0$ or vice-versa. The parameters correspond to the ones in Table 1 for the populations $P_1 := P_1^2$ and $P_2 = P_2^4$. The simulations with $C_1(0) = 0.1$ and $C_2(0) = 0$ corresponds, respectively, to the case where the fake news starts to spread in population 1 (the population with the highest index of human development among the populations). The other case is the opposite. In both cases, we have no control, i.e. $\xi_1 = \xi_2 = 0$.

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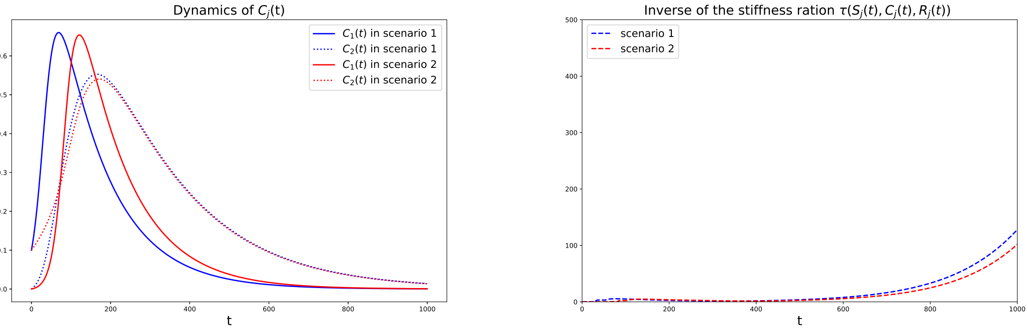

Figure 1: Simulations using the parameters in Table 1 for the populations $P_1 := P_1^2$ and $P_2 = P_2^4$, initial conditions with $C_1(0) = 0.1$ and $C_2(0) = 0$ correspond to "scenario 1", and $C_1(0) = 0$ and $C_2(0) = 0.1$ correspond to "scenario 2".

The dynamics of the contaminated $C_j(t)$ of both populations in the simulated scenario 1 are depicted in Figure 1. Since $R_i(t) = \gamma_j \int_0^t C_j(s)ds$, its dynamics can be deduced. The presented results in Figure 1-(left) suggests that the impact of the fake news started in the higher or lower developed populations implies a delay in the dynamics of $C_j(t)$ but with little impact in the pick of contamination. Indeed, 0.6% and 1.2% in populations 1 and 2, respectively. In the Figure 1-(right), we present the results for $\tau(t, U(t))$. Figure 1-(right) suggests that, for non-homogeneous populations (differently of the case analyzed in [2, 4]), large values for $\tau(t, U(t))$ are obtained depending on where the diseases started (in a more developed population or not), but have no significant impact on the amount of contaminated individuals.

Simulated Scenario 2: In this scenario, we analyze the case where there is a control ($\xi_1 = \xi_2 = 1$) with an efficacy of $\rho_1 = \rho_1 = 0.1$ applied to both populations. The remaining parameters are the same as in Simulated Scenario 1.

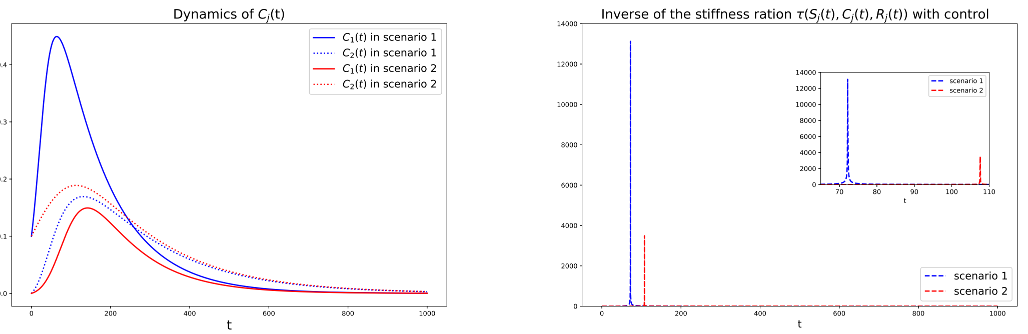

Figure 2: Simulations using the parameters in Table 1 for the populations $P_1 := P_1^2$ and $P_2 = P_2^4$, initial conditions with $C_1(0) = 0.1$ and $C_2(0) = 0$ correspond to "scenario 1", and $C_1(0) = 0$ and $C_2(0) = 0.1$ corresponds to "scenario 2" and a symmetric control ($\xi_1 = \xi_2 = 1$) with an efficacy of $\rho_1 = \rho_2 = 0.01$ in both populations.

The dynamics of the contaminated $C_j(t)$ of both populations in the simulated scenario 2 are depicted in Figure 2. Since $R_i(t) = \int_0^t \gamma_i C_j(s) + \rho_i S_i(s)ds$, its dynamics can be deduced. The presented results in Figure 2-(left) show that even a little control over the information has a large impact on the dynamics of the fake news spreading (compare with the Simulated Scenario 1 in Figure 1). In particular, the large impact is in the contamination of the population P_2 if the fake news starts in the population P_1, or in both populations in the case of the fake news starting in the less developed population. In Figure 2-(right), we can see two spikes with distinct intensities for $\tau(t, U(t))$, depending on each of the simulated scenarios. It suggests that the control strategy is more efficient in containing the fake news diffusion between the populations at the lower spike level.

Simulated Scenario 3: Here we analyze the impact of the HDI and IPI indices, which correspond to the parameters in Table 1 on the diffusion of fake news. In the simulations, the fake news always started in the population with a higher HDI ($I_1(0) = 0.1$ and $I_2(0) = 0$), and no control was used ($\xi_1 = \xi_2 = 0$).

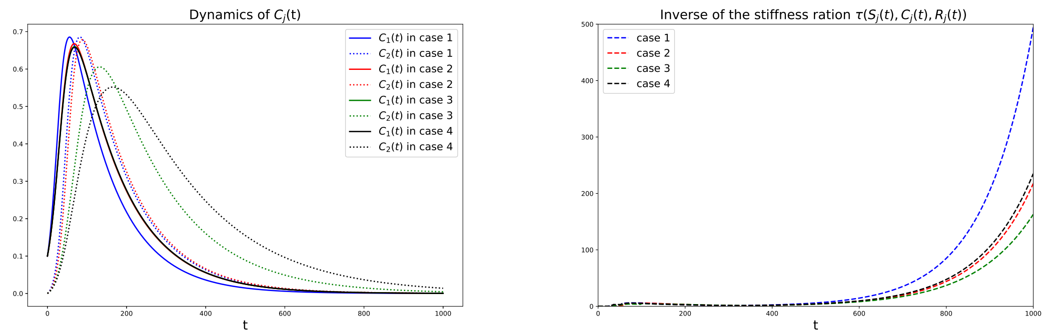

Figure 3: Simulated scenarios with interacting populations have distinct development indices according to Table 1. The case studies are: Case 1: $P_1^1 \times P_2^2$, Case 2: $P_1^2 \times P_1^2$, Case 3: $P_1^2 \times P_2^3$ and Case 4: $P_1^2 \times P_2^4$.

The dynamics of the contaminated $C_j(t)$ in both populations in the simulated scenario 3 are depicted in Figure 3. Since $R_i(t) = \int_0^t \gamma_i C_j(s) ds$, its dynamics can be deduced. The presented results in Figure 3-(left) show that the impact of the fake news spreading from the population with a higher (P_1) to a lower (P_2) index of development is monotonically decreasing. This is probably due to the IPI of the population, P_2. In Figure 3-(right), we show the corresponding evolution of $\tau(t, U(t))$. Figure 3-(right) suggests that for a homogeneous or near-homogeneous population that interacts, the larger the values of $\tau(t, U(t))$ the faster is the diffusion of the fake news and, consequently, the reestablishment of the true, as concluded by [2, 4]). On the other hand, for two higher non-homogeneous interacting populations, the values of $\tau(t, U(t))$ are compared with the values of the homogeneous case (see Case 2 and Case 4) but the fake news is slowly diffused in the population 2. Therefore, the values of $\tau(t, U(t))$ as a measure of the velocity of the fake news diffusion shall be revisited for interacting populations.

4 Conclusions and future directions

In this contribution, we propose a reinterpretation of a compartmental SIR-type model to describe the dynamics of the dissemination behavior of fake news among two distinct sub-populations that share information. The obtained results from the numerically simulated scenarios suggest that

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the velocity of fake news diffusion among the populations is largely impacted by the gap between the human development indices of each population. They also show that a small percentage of control over the information shared by the population leads to a large decrease in the amount and velocity of fake news diffusion. As a consequence, the inverse of the stiffness ratio is not monotonic among largely inhomogeneous interacting sub-populations, which affects the speed of the spread of fake news, contrary to the conclusions in [2, 4].

The obtained results will be complemented in future contributions with the stability and bifurcation analysis.

The ideas presented in this paper are naturally extended for populations that interact in a network [5, 7].

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