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A nonparametric Bayesian model for prediction of solar power generation

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In the last few years, there happened a significant increasing of solar energy generation so as by photovoltaics plants as by residences with board photovoltaic instaled on the roofs. This is mainly due to intereset of the society and governments for clean and renewable energies that reduces the amount of $C0_2$ emission. Do to this, to each day more photovoltaic plants are being connected to the electrical systems of the cities. However, according [1] and [2], this may causes instability to the grid, making it the greatest challenge to the industry.

A way to approach this issue is by modeling the solar energy generation and developing strategies for forecasting the solar energy that will be generated in the next few days. In this way, be able to balance the generated energy with the demand for electricity in the cities through efficient grid transmission with low wasting and low cost. Under this scenery, we propose a nonparametric Bayesian approach for modeling solar energy and making predictions.

Since the solar power generated in a day present a nonlinear unstable behaviour, our approach is based on the assumption that the recorded values are around a nonlinear continuous function $f(t)$ that indicates the tendency of the values, where t denotes the time instant of the day that measure were recorded. That is, we assume that the solar power generated Y_t is modelled by the following additive model $Y = f(t) + \varepsilon$, where $f(t)$ is nonlinear function and ε is a random error. However, in opposite to setting up $f(t)$ as a known mathematical function with a fixed number of parameters, such as a polynomial function, we consider $f(t)$ to be an unknown quantity that must be estimated from observed data.

Then, in order to make inference on $f(\cdot)$, we assume that *a priori* a n-dimensional vector of function values evaluated at n points $\mathbf{f} = (f(1), \ldots, f(n))$ is generated according to a Gaussian process. This means that *a priori* we are estimating $f(\cdot)$ by "smooth functions" obtained by the generation of values of a multivariate normal distribution and linking the generated points by lines. To complete our Bayesian approach, we put prior distributions for the parameters of the Gaussian process and for the variance of the random errors in order to get the following hierarchical model

$$\begin{aligned} \mathbf{Y} = (Y_1, \ldots, Y_n)\,|\mathbf{f}, \sigma^2 &\sim \mathcal{N}_n\left(\mathbf{f}, \sigma^2\mathbb{I}\right); \qquad (1)\\ \mathbf{f} = (f(1), \ldots, f(n))\,|\Sigma_{\mathbf{f}} &\sim \mathcal{GP}(\mathbf{0}, \Sigma_{\mathbf{f}}) \\ \sigma^2 &\sim \mathcal{IG}\left(\frac{a}{2}, \frac{b}{2}\right); \\ \Sigma_{\mathbf{f}}|\gamma, \sigma^2_{\mathbf{f}}, \nu &\sim \mathcal{IW}\left(\gamma, \sigma^2_{\mathbf{f}}\mathbb{A}_\nu\right); \end{aligned}$$

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where $\mathcal{N}_n(\cdot)$ represents the n-variate Gaussian distribution, $\mathcal{GP}(\cdot)$ represents a Gaussian process, $\mathcal{IG}(\cdot)$ represents the inverse Gamma distribution and $\mathcal{IW}\left(\gamma, \sigma^2_{\mathbf{f}}\mathbb{A}_\nu\right)$ represents the inverse-Wishart distribution with parameters γ and $\sigma^2_{\mathbf{f}}\mathbb{A}_\nu$, being $\mathbb{A}_\nu$ a matrix of dimension $n \times n$ with elements $k(t, t')$, $\mathbb{I}$ is the identity matrix of dimension n and constants a, b, γ and $\sigma^2_{\mathbf{f}}$ are known hiperparameters, for $t, t' = 1, \ldots, n$. In addition, we assume that each term $k(t, t')$ is calculated according to the squared exponential kernel, *i.e.*,

$$k(t, t') = \sigma^2_{\mathbf{f}_d} \exp\left\{-\frac{(t - t')^2}{2\nu^2}\right\}, \tag{2}$$

for $t, t' = 1, \ldots, n$. We fix $a = b = 0.1$, $\gamma = 1$ and $\sigma^2_{\mathbf{f}} = 100$ in order to get noninformative prior distributions. To complete this hierarchical Bayesian model, we point out some reasons that led us to consider this structure of prior distributions. The option for the Inverse-Gamma distribution for σ^2 lies in the fact of this is a natural conjugated prior. Analogously, the inverse Wishart distribution is the natural conjugated prior for $\Sigma_{\mathbf{f}}$.

Since the joint posterior distribution for the parameters of interest does not have a known mathematical form, we got the conditional posterior distributions and developed an MCMC algorithm to get the parameter estimates. The MCMC algorithm is a Gibbs sampling algorithm that iteratively generates values from the following conditional posterior distributions

$$\mathbf{f}|\mathbf{y}, \Sigma, \Sigma_{\mathbf{f}} \sim \mathcal{N}_n\left(\Sigma^{-1}\left(\Sigma^{-1} + \Sigma_{\mathbf{f}}^{-1}\right)\mathbf{y}, \left(\Sigma^{-1} + \Sigma_{\mathbf{f}}^{-1}\right)^{-1}\right) \tag{3}$$

$$\sigma^2|\mathbf{y}, \mathbf{f}, a, b \sim \mathcal{IG}\left(\frac{a+n}{2}, \frac{b + (\mathbf{y} - \mathbf{f})^T(\mathbf{y} - \mathbf{f})}{2}\right) \tag{4}$$

$$\Sigma_{\mathbf{f}}|\mathbf{f}, \sigma^2_{\mathbf{f}}\mathbb{A}_\nu, \gamma \sim \mathcal{IW}(\gamma + n + 1, \mathbf{f}\mathbf{f}^T + \sigma^2_{\mathbf{f}}, \mathbb{A}_\nu) \tag{5}$$

The performance of the proposed modeling is illustrated in a case study, in which, in which, we model the solar energy generated by a plant installed in the location (-20.510867, -54.619882), Faculty of Veterinary Medicine and Animal Science (FAMEZ) of the Federal university of Mato Grosso do Sul, in a period of $d = 100$ days. As result, the proposed approach has presented an average mean square error (MSE) of 0.0005 in relation to the estimated values, and an average MSE value of 0.0004 in relation to the predicted values.

Three advantages of the proposed modeling are:

(i) the modeling is very flexible and adapts to the number of recorded values in a day;

(ii) The fitted model is obtained in a direct way and does not need the development of model comparison, as is usually done when a parametric approach is considered;

(iii) The inference process is done based on a Metropolis-Hastings within Gibbs sampling algorithm that can be easily implemented in free software like R or WinBUGS.

Referências

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