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Real Time Manual Control of Fractional Punishment in Optional Public Goods Game

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Interpreting the results of simulating a non linear system of differential equations requires a great deal of background knowledge about the system that is being analyzed. If a controller is implemented, the dynamics of the system may change completely, creating another barrier in the time that is needed to analyze the system.

In this work we seek to implement a circuit that may emulate the behaviour of a non linear system, subject to a controller which may be manually set at different values by the user in real time, making it easier to get quick results that could later be verified by more thorough analysis.

The testing will be done in an Optional Public Goods Game (OPGG) [1, 2], which models the cooperation, defection and independence of a society with respect to a certain public good. In such system the ideal state will be for all of society to cooperate, and to obtain such result fractional punishment (sanctioning a fraction of free riders in the system) has been shown to be an effective, yet less expensive tool for achieving such goal [3].

To this end, let us define our system in the following way, let (t_0, t_f) denote a time interval, $w(t)$ the state variable and $\mathcal{W} \subset \mathbb{R}^3$ the state space. We consider $\mathcal{W} = \mathcal{S}_3$ defined as $\mathcal{S}_3 = \left\{[x, y, z]^T \in \mathbb{R}^3 : x, y, z \geq 0 \text{ and } x + y + z = 1\right\}$. Given an initial condition $w(0) \in \mathcal{W}$ the state equation has the form [3]:

$$\dot{x} = x\,(p_x(w) - \bar{p}(w, v)) \tag{1}$$
$$\dot{y} = y\,(p_y(w, v) - \bar{p}(w, v)) \tag{2}$$
$$\dot{z} = z\,(p_z - \bar{p}(w, v)) \tag{3}$$

where $w(t) = [x(t), y(t), z(t)]^T$ such that each entry $0 \leq x(t), y(t), z(t) \leq 1$ is the frequency of each of the corresponding available strategies of the population at a specific time t (cooperators, defectors and loners, respectively). Each p_i is the payoff of the ith strategy, and $\bar{p}$ is its average given by $\bar{p} = xp_x + yp_y + zp_z$. The distributed control is denoted by $v \in [0, 1] = \mathcal{U}$. We indicate the dependence of w on $v \in \mathcal{U}$ using the notation $w(v)$.

It has been shown in [3] that for the given system, a high enough fractional punishment v makes state $w^* = [1, 0, 0]^T$ an attractor, solving the problem of cooperation and adding a challenge in maintaining the sanctioning system.

Our circuit will emulate the open loop system in a microcontroller and let us change the fraction of punished free riders with the use of a potentiometer and an analog input voltage, while at the same time outputting the three states as analog voltage signals with the help of a digital to analog converter (DAC). This signals could then be visualized in real time with the help of an osciloscope, as seen in figure 1.

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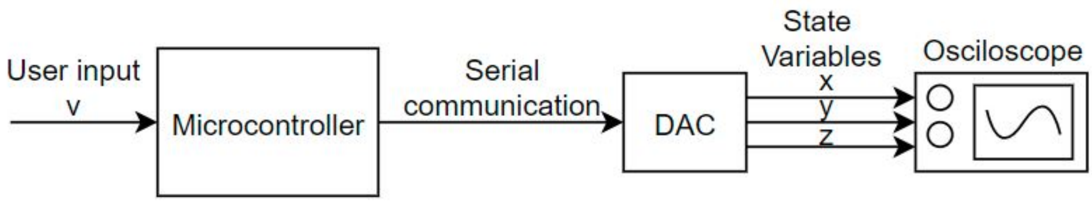

Figure 1: Schematic diagram of the circuit to be implemented. Source: elaborated by the author.

A discretization of the system that solves for the states using the classic Runge-Kutta algorithm has been implemented in Matlab based on the works of [4]. Preliminary results of this process replicate [3] and [5] to a high degree of accuracy. With this in mind the implementation in a C language based microcontroller is already seen as feasible, even with the complexity of the system.

The results of this work give us a practical tool that helps in the analysis of optimal controllers for fractional punishment, furthering the understanding of previous works [5].

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