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Mimetic Operator Discretization 1D: Exploration of classical iterative methods and preconditioners

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This work performs experiments for the numerical resolution of the one-dimensional Poisson equation with Robin boundary conditions, using the second-order mimetic discretization method, based on the 1D Castillo-Grone mimetic operator [1]. Consider the following equation on a uniform grid,

$$\nabla^2 u(x) = f(x) \quad on \quad [0,1] \tag{1}$$

subject to Robin boundary conditions

$$\alpha f(0) - \beta f'(0) = -1; \;\; \alpha f(1) + \beta f'(1) = 0,$$

where

$$f(x) = \frac{\lambda^2 e^{\lambda x}}{e^{\lambda x} - 1}, \;\; \alpha = -e^{\lambda}, \;\; \beta = \frac{e^{\lambda} - 1}{\lambda}, \;\; \lambda = -1.$$

To solve the linear system $Au = f$ associated with this problem, we used the restarted GMRES with $m = 10$, and the BiCGStab as iterative methods. Also, the Jacobi and SOR with $\omega = 1$ are used to try to improve the convergence. In Tables 1 and 2, colum h corresponds to step size of the uniform grid, $Condest$ is the condition number of the preconditioned matrix of the linear system. For each value of h we find the relative error $||u - u^*||/||u^*||$, where u is the approximate numerical solution by the selected iterative method plus the preconditioner and u^* is the analytical or exact solution.

Table 1: Numerical results for $||u - u^*||/||u^*||$ with Jacobi Preconditioner

h	Condest	GMRES(10)	BiCGStab
0.20	311.9	0.002184	0.002184
0.10	1073.0	0.000537	0.000545
0.05	3948.4	0.000126	0.00136
0.01	91903.2	1.165131	0.000004

Table 2: Numerical results for $||u - u^*||/||u^*||$ with SOR($\omega = 1$) Preconditioner

h	Condest	GMRES(10)	BiCGStab
0.20	190.6	0.002184	0.002184
0.10	662.5	0.000543	0.000541
0.05	2421.5	1.008095	0.00132
0.01	55928.2	1.142317	0.000010

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The convergence of the GMRES(10) is only achieved up to $h = 0.05$ using Jacobi and $h = 0.1$ using SOR, for smaller values of h the GMRES(10) diverges. Conversely, for BiCGStab the convergence is maintained up to $h = 0.001$, but for smaller values, the BiCGStab also diverges. In Figure 1 we can see the performance of the residual norm for the different combinations of iterative methods and preconditioners. The BiCGStab method has a bad performance at the beginning, even with a growth of the residual norm, but then a rapid decrease is achieved. Òn the other hand, the restarted GMRES is characterized by its non-increasing monotonic norm, but for this problem only the preconditioning with SOR achieves convergence.

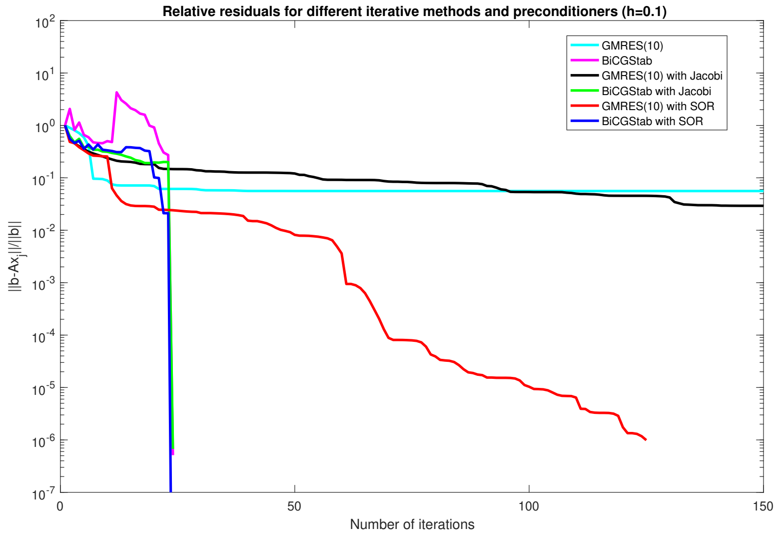

Figure 1: Relative residuals for GMRES(10) and BiCGStab using step size $h = 0.1$.

By decreasing h, only an appropriate preconditioner will allow the iterative method to achieve convergence. However, the choice of such preconditioning technique is not clear. In this work we will look for preconditioning and iterative methods techniques that overcome stagnation by reducing the relative error to less than 10^{-6} for $h < 0.001$. Adaptive-parameter iterative methods [2] and other preconditioners [3] will be tested for such goal.

References

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