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Alternative Approach for Deriving an Unconventional Distance Protection Algorithm

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Abstract. This work presents an alternative approach for deriving an unconventional distance protection algorithm suitable for long single-circuit transmission lines. Such approach is based on the application of Kirchhoff's laws to a system modeled in the frequency domain. The electric power system considered and the steps of the protection algorithm deduction approach are detailed. As a result, the same input signals from the literature are obtained, which demonstrates the adequacy of such methodology to different systems.

Keywords. Alternative Mathematical Approach, Distance Protection Algorithm, Electric Power Systems, Long Transmission Lines

1 Introduction

Conventional distance protection algorithm is derived from a transmission line (TL) model with lumped parameters, obtained by multiplying the series impedance per unit length by the total length, disregarding propagation effects and the transverse capacitance of the TL [6]. Such approximation is adequate for either short or medium TLs, since in these there is an almost linear relationship between the positive-sequence impedance to the fault and the distance to the fault. However, it can lead to significant errors in the estimation of apparent impedances for long-distance faults in very long TLs (with extension greater than approximately 400 km), since in these cases the capacitive effect is accentuated. Thus, it can cause the distance relay to underreach, that is, not to trip in cases where the fault occurs within the protected zone.

Given this, Xu et al. [7] proposed an unconventional distance protection algorithm, derived from a TL model with distributed parameters suitable for long single-circuit TLs, which demonstrated satisfactory applicability in a 645 km long single-circuit TL. In [7], the mathematical path adopted by the authors for the deduction of the proposed algorithm is demonstrated, which is later cited in [1].

The objective of this work is to present in detail and show the extensiveness of an alternative mathematical approach for the deduction of the same unconventional distance protection algorithm proposed by Xu et al. [7], concerning the input signals for the ground distance elements. Such alternative approach is based on the analysis of electrical power systems in the frequency domain. It was developed to enable the deduction of protection algorithms suitable for balanced long parallel TLs either connected to common buses [3] or with no common bus [4], which had [7] as a starting point.

A preprint has previously been published [2].

This work is structured in three sections, including this introduction. In the second section, the modeled electric power system and the alternative approach for the derivation of the unconventional distance protection algorithm are described in detail. The third section concludes this work.

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2 Alternative Approach

Figure 1 illustrates the connection of the positive- (+), negative- (-) and zero-sequence (0) networks for an A-phase-to-ground (AG) fault applied at node F of a balanced single-circuit TL, modeled as a cascade of π-equivalent circuits.

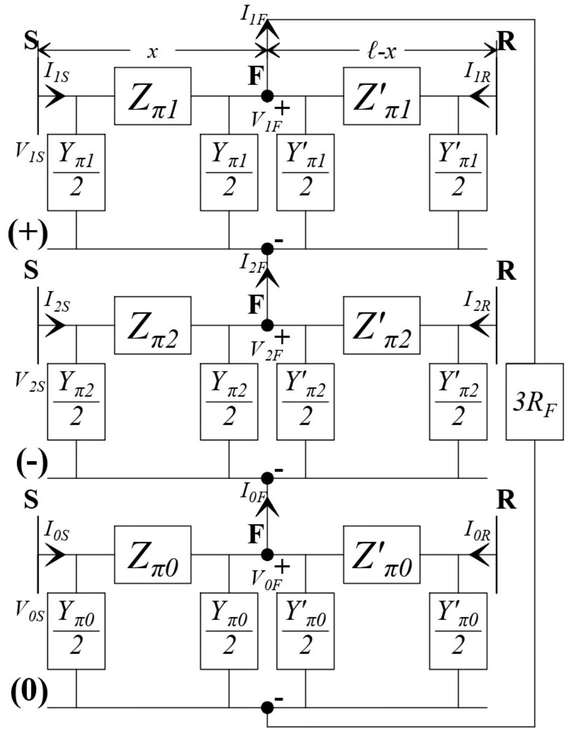

Figure 1: Configuration of sequence diagrams for AG fault applied to a single-circuit TL. Source: Figure created by author.

The equivalent series impedance (Z_π) and equivalent shunt admittance (Y_π) parameters were defined, respectively, according to [5]:

$$Z_{\pi i} = Z_{Ci} sinh(\gamma_i x) = \sqrt{\frac{Z_i}{Y_i}} sinh(\sqrt{Z_i Y_i} x) \tag{1}$$

$$\frac{Y_{\pi i}}{2} = \frac{1}{Z_{Ci}} tanh\left(\frac{\gamma_i x}{2}\right) \tag{2}$$

where i is the index of the symmetrical component (i = 1, 2, 0, respectively for the positive-, negative-, and zero- sequence), x is the distance between the relay installation point and the fault application point, Z_C is the characteristic impedance, γ is the propagation constant, Z is the series impedance per unit length, and Y is the shunt admittance per unit length. Furthermore, in Figure 1, ℓ is the total length of the TL, S and R represent the TL sending and receiving ends, respectively.

The A-phase voltage (V_a) at the relay bus (S bus) is given by:

$$V_{aS} = V_{0S} + V_{1S} + V_{2S} \tag{3}$$

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in which, by applying Kirchhoff's current (I) and voltage (V) Laws, the sequence voltages can be described by:

$$V_{0S} = Z_{\pi 0}\left(I_{0S} - \frac{Y_{\pi 0}}{2}V_{0S}\right) + V_{0F} = Z_{\pi 0}I_{0S} - Z_{\pi 0}\frac{Y_{\pi 0}}{2}V_{0S} + V_{0F} \tag{4}$$

$$V_{1S} = Z_{\pi 1}\left(I_{1S} - \frac{Y_{\pi 1}}{2}V_{1S}\right) + V_{1F} = Z_{\pi 1}I_{1S} - Z_{\pi 1}\frac{Y_{\pi 1}}{2}V_{1S} + V_{1F} \tag{5}$$

$$V_{2S} = Z_{\pi 1}\left(I_{2S} - \frac{Y_{\pi 1}}{2}V_{2S}\right) + V_{2F} = Z_{\pi 1}I_{2S} - Z_{\pi 1}\frac{Y_{\pi 1}}{2}V_{2S} + V_{2F} \tag{6}$$

Permuting $Z_{\pi i}$ and $Y_{\pi i}$ by their expressions in hyperbolic form according to (1) and (2), respectively, the product of the first by half of the second is given by:

$$Z_{\pi i}\frac{Y_{\pi i}}{2} = sinh\left(\gamma_i x\right) tanh\left(\frac{\gamma_i x}{2}\right) \tag{7}$$

Considering u equal to the argument of the hyperbolic tangent function in (7), using the definition of such function and the following hyperbolic identity:

$$sinh\left(2u\right) = 2sinh\left(u\right) cosh\left(u\right) \tag{8}$$

it results for (7) that:

$$Z_{\pi i}\frac{Y_{\pi i}}{2} = 2sinh^2\left(\frac{\gamma_i x}{2}\right) \tag{9}$$

Considering then v equal to the double of u and applying the following hyperbolic identity:

$$sinh^2\left(\frac{v}{2}\right) = \frac{cosh\left(v\right) - 1}{2} \tag{10}$$

it gets to:

$$Z_{\pi i}\frac{Y_{\pi i}}{2} = cosh\left(\gamma_i x\right) - 1 \tag{11}$$

Taking into account (11), equations (4), (5), and (6) can be expressed in the form:

$$V_{0S} = Z_{C0}sinh\left(\gamma_0 x\right) I_{0S} - cosh\left(\gamma_0 x\right) V_{0S} + V_{0S} + V_{0F} \tag{12}$$

$$V_{1S} = Z_{C1}sinh\left(\gamma_1 x\right) I_{1S} - cosh\left(\gamma_1 x\right) V_{1S} + V_{1S} + V_{1F} \tag{13}$$

$$V_{2S} = Z_{C1}sinh\left(\gamma_1 x\right) I_{2S} - cosh\left(\gamma_1 x\right) V_{2S} + V_{2S} + V_{2F} \tag{14}$$

Substituting in (3) the formulations obtained in (12), (13) and (14), by rearranging the terms, it is obtained:

$$\begin{aligned} V_{aS} - V_{0S} - V_{1S} - V_{2S} \\ + cosh\left(\gamma_0 x\right) V_{0S} + cosh\left(\gamma_1 x\right) V_{1S} + cosh\left(\gamma_1 x\right) V_{2S} \\ = Z_{C0}sinh\left(\gamma_0 x\right) I_{0S} + Z_{C1}sinh\left(\gamma_1 x\right) I_{1S} + Z_{C1}sinh\left(\gamma_1 x\right) I_{2S} \\ + V_{0F} + V_{1F} + V_{2F} \end{aligned} \tag{15}$$

For a solid AG fault, that is, with fault resistance (R_F) equal to zero, taking into account Figure 1:

$$V_{0F} + V_{1F} + V_{2F} = V_{aF} = R_F I_{aF} = 0 \tag{16}$$

Defining, for convenience:

$$A_1 = cosh\left(\gamma_1 x\right) \tag{17}$$

$$A_{V0} = A_1 V_{0S} \tag{18}$$

$$B_1 = Z_{C1} sinh\left(\gamma_1 x\right) \tag{19}$$

$$B_{I0} = B_1 I_{0S} \tag{20}$$

Therefore, admitting (16), canceling V_{aS} taking into account (3), adding and subtracting (18) and (20), it results for (15) that:

$$\begin{aligned} cosh\left(\gamma_1 x\right) V_{0S} + cosh\left(\gamma_1 x\right) V_{1S} + cosh\left(\gamma_1 x\right) V_{2S} \\ + cosh\left(\gamma_0 x\right) V_{0S} - cosh\left(\gamma_1 x\right) V_{0S} \\ = Z_{C1} sinh\left(\gamma_1 x\right) I_{0S} + Z_{C1} sinh\left(\gamma_1 x\right) I_{1S} + Z_{C1} sinh\left(\gamma_1 x\right) I_{2S} \\ + Z_{C0} sinh\left(\gamma_0 x\right) I_{0S} - Z_{C1} sinh\left(\gamma_1 x\right) I_{0S} \end{aligned} \tag{21}$$

Putting V_{0S}, I_{0S}, A_1, and B_1 in evidence:

$$\begin{aligned} cosh\left(\gamma_1 x\right)\left[V_{0S} + V_{1S} + V_{2S} + \frac{cosh\left(\gamma_0 x\right) - cosh\left(\gamma_1 x\right)}{cosh\left(\gamma_1 x\right)} V_{0S}\right] \\ = Z_{C1} sinh\left(\gamma_1 x\right)\left[I_{0S} + I_{1S} + I_{2S} + \frac{Z_{C0} sinh\left(\gamma_0 x\right) - Z_{C1} sinh\left(\gamma_1 x\right)}{Z_{C1} sinh\left(\gamma_1 x\right)} I_{0S}\right] \end{aligned} \tag{22}$$

Simplifying the V_{aS} and I_{aS} expressions, see (3), and defining the voltage and current zero-sequence compensation parameters, k_V and k_I, respectively, (22) can be reduced to:

$$cosh\left(\gamma_1 x\right)\left(V_{aS} + k_V V_{0S}\right) = Z_{C1} sinh\left(\gamma_1 x\right)\left(I_{aS} + k_I I_{0S}\right) \tag{23}$$

where

$$k_v = \frac{cosh\left(\gamma_0 x\right) - cosh\left(\gamma_1 x\right)}{cosh\left(\gamma_1 x\right)} \tag{24}$$

$$k_I = \frac{Z_{C0} sinh\left(\gamma_0 x\right) - Z_{C1} sinh\left(\gamma_1 x\right)}{Z_{C1} sinh\left(\gamma_1 x\right)} \tag{25}$$

Finally, the apparent impedance (Z_R) seen by the AG unit of the unconventional distance relay, obtained by the ratio between the voltage input signal (V_R) and the current input signal (I_R), is as follows:

$$Z_R = \frac{V_R}{I_R} = \frac{V_{aS} + k_V V_{0S}}{I_{aS} + k_I I_{0S}} = \frac{Z_{C1} sinh\left(\gamma_1 x\right)}{cosh\left(\gamma_1 x\right)} = Z_{C1} tanh\left(\gamma_1 x\right) \tag{26}$$

In order for Z_R to correspond to the positive-sequence impedance to the fault, the following filter is applied, as proposed in [7]:

$$Z_R^* = \frac{Z_1}{\gamma_1} tanh^{-1}\left(\frac{Z_R}{Z_{C1}}\right) = xZ_1 \tag{27}$$

where Z_R^* is the corrected apparent impedance seen by the unconventional distance relay.

3 Conclusions

This work demonstrated the extensiveness of an alternative approach for the derivation of unconventional distance protection algorithms. By means of alternative mathematical methods, the same formulations proposed in the literature were achieved for the system considered. Therefore, such methodology proved to be a viable alternative for the deduction of new protection algorithms in electrical power systems with peculiar configurations.

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