Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 11, n. 1, 2025.

Trabalho apresentado no XLIII CNMAC, Centro de Convenções do Armação Resort - Porto de Galinhas - PE, 2024

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Mixed-integer Programming Formulations for the Optimisation of Multi-directional Petroleum Exploration

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Abstract. Directional drilling can be seen as a well-established technology that has advanced in the past few years, allowing higher productivity in petroleum exploration. On the other hand, the design of optimal multi-directional drilling paths has not gained much attention in the literature. Existing models and algorithms for the optimisation of directional drilling paths often disregard important petrophysical attributes or apply heuristic methods, which cannot guarantee the optimality of the generated solutions. In this paper, we employ mixed-integer programming (MIP) to optimize multiple directional drilling paths. This is achieved by integrating a screening step, capable of identifying the most promising target regions in terms of flow capacity, with an MIP model responsible for selecting and sequencing the identified targets. Our approach considers several constraints such as drift angles, maximum wellbore length, and minimum safety distances. In addition, a branch-and-cut algorithm is proposed for solving real-world multi-directional instances of challenging sizes. We carry out a case study in the Campos Basin to validate the proposed models. Preliminary results show that the optimized drilling paths have superior performance when compared to the historical average recovery factor of the Campos Basin.

Keywords. Exploration and Production, Well Drilling, Hydraulic Flow Units, Mixed-integer Programming.

1 Introduction

In petroleum exploration engineering, directional wells consist of wells with a borehole that deviates from purely vertical and horizontal straight lines [15]. The azimuth angle of directional wells typically deviates from its central axis by an angle between 20 and 80 degrees. Among the existing four types of wells, namely, vertical, horizontal, directional and multilateral, directional wells effectively serve as an umbrella category that includes all other types. The design of directional wells requires careful path planning, with a precise specification of target areas, taking into account the available geosteering equipment. Such paths must usually consider operational constraints, such as turning angles, maximum length and safety distances [7]. The problem becomes even more challenging if multiple directional well paths must be planned simultaneously. In this case, a careful analysis of all possible scenarios is of primordial importance.

When examining all Brazilian oil basins, the average global recovery rate was approximately 23.6% in 2022. In particular, the *Campos Basin*, the main O&G production field in Brazil, had a recovery rate of around 15.4%, which is low relative to the volume of oil available. Among many

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reasons that could justify such low performance, it has been argued that the O&G industry usually lacks strategies that integrate the placement of wells, identification of prime drilling locations, and optimisation of directional drilling paths, especially in offshore deep-sea settings [10].

Even though literature can be found on the optimisation of drilling paths, most of these studies insufficiently account for petrophysical factors, which are crucial in the preliminary selection of drilling targets [2]. Several heuristic techniques have been introduced to devise the most efficient drilling paths from an origin point to a specified destination. For example, one can cite, Particle Swarm Optimisation (PSO) [4], Population-based Algorithms [9] and sequential gradient restoration [11]. However, the literature is scarce if one needs to optimize various paths over several target locations within an oil field.

In this work, we explore an alternative method to evaluate the performance of directional wells in the Campos Basin, comprising two primary phases: (i) a target screening process that identifies regions of maximum potential flow, and; (ii) two mixed-integer programming (MIP) formulations for the selection and sequencing of targets identified in the previous step. The first formulation, which is compact, requires a preprocessing step and aims at maximising a score based on the quality of targets. The second one, a cut-based model designed for real-world scenarios, introduces safety distance constraints iteratively. Both models account for factors like drift angles, wellbore length limits, and safety margins. We tested our methods using the UNISIM-I dataset from the *Namorado* sandstone oilfield in the Campos Basin, Brazil. Our best-performing well simulation showed a recovery rate of over 30%, surpassing the historical average for the reservoir in question.

2 Description and Discretisation of UNISIM-I

The UNISIM-I-D model is a well-known representation of the Namorado formation that has been extensively studied in the literature. We refer the interested reader to, e.g., [5, 7] for further information about this model.

Hydraulic Flow Units (HFUs) represent specific regions within a reservoir that exhibit distinctive characteristics. For example, [17] describes an HFU as a particular volume within the reservoir that is correlative, mappable, and identifiable on wireline logs, with the capability for communication with other HFUs defined similarly. Flow units can be understood as a collection of adjacent cells within a 3D corner-point grid. Each cell c in the model is identified by a set of logical indices, defining the oilfield domain as the set

$$\overline{\Omega} = \{c_{(i,j,k)};\ 1 \le i \le I, 1 \le j \le J, 1 \le k \le K\},\ I, J, K \in \mathbb{Z}_+^*, \tag{1}$$

encompassing a total number of n_p active cells. In this context, $\overline{\Omega}$ represents the discrete embodiment of an entire oilfield.

The screening process for directional drilling can be summarized in the following steps. The approach begins by identifying clusters based on Discrete Rock Types (DRT), which represent distinct volumes of rock with similar petrophysical properties. Subsequently, each cluster is mapped to a graph through a one-to-one function, transforming the cluster cells into graph nodes. The third step involves calculating the closeness centrality for each node in the graph [7].

Finally, the "maximum closeness centrality" (MCC) cell within each cluster is determined based on the most central node of the graph. These MCC cells identified through the reservoir model, referred to as possible drilling targets, can be ordered and represented by the following set:

$$T = \{T_1, T_2, \ldots, T_n\}, \tag{2}$$

where n is the total number of MCC cells identified. For more details, see [7].

3 Mixed-integer Programming Formulations

In this paper, we consider the problem of designing a predefined number of directional drilling paths passing through the most promising target cells in a reservoir, subject to several operational constraints, such as maximum wellbore length, drift angle, and safety distances. We chose to model this problem as a team orienteering problem (TOP), a well-studied variant of the VRP [3], which is, in turn, one of the most famous in the literature of combinatorial optimisation and MIP.

The problem is defined over a graph $G = (V, A)$, in which $V = \{0, \ldots, n-1\}$ is a set consisting of a dummy vertex 0 and set of $n-1$ MCC cells T_i, $i \in T$ (Eq. (2)), with coordinates $\boldsymbol{x}_i = (x_i, y_i, z_i)$. Following our previous work [7], to each cell $i \in V$ we associate a cell-weighting parameter p_i, the so-called **prize**, computed as the Euclidean distance between the centroids of i and its nearest cell located at the boundary of the reservoir. Therefore, the higher the value of p_i, cell i will be given higher priority. In other words, among all MCC cells, the inner ones must be preferred over the more external ones in the reservoir.

The arc set $A = \{(i, j) : \alpha_{ij} \le \alpha, z_i > z_j, \forall i, j \in V \setminus \{0\}\}$ represents the connections between any two cells i and j in V, such that the maximum drift angle of arc (i, j), denoted by α_{ij}, is smaller than a given value α and the vertical position of cell i is higher than the vertical position of cell j. Therefore, graph G is constructed in such a way as to incorporate operational wellbore constraints such as enforcing monotonically descending paths and maximum drift angles. Let us also define the set $K = \{0, \ldots, m-1\}$ to represent the set of drilling *paths* $r_k = (i_0, i_1, \ldots, i_q)_k, i \in V, q < n, k \in K$, such that for any $k^1, k^2 \in K, r_{k^1} \cap r_{k^2} = \varnothing$. A minimum safety distance d^s between any two paths must be enforced to guarantee a safe and efficient operation. In this paper, the distance $d(r_{k^1}, r_{k^2})$ between paths r_{k^1} and r_{k^2} is computed as the minimum distance between their nearest arcs $(i, j) \in r_{k^1}$ and $(l, k) \in r_{k^2}$. We say that a *collision* occurs if any two paths r_{k^1} and r_{k^2} violate this constraint, i.e., $d(r_{k^1}, r_{k^2}) < d^s$. Finally, we define an auxiliary tuple set $\bar{A} = \{(i, j, l, m) : \|(i, j), (l, m)\| < d^s, (i, j), (l, k) \in A\}$, in which $\|.\|$ denotes the minimum distance between any two arcs. Therefore, set $\bar{A}$ represents the set of all collisions among the arcs of set A. We highlight that building this set requires the computation of $\mathcal{O}(n^4)$ distances.

Following the above definitions, the problem can be formally defined as computing m paths over graph G that maximise the total collected prize $\sum_{k \in K} \sum_{i \in r_k} p_i$ of the visited cells, subject to $\sum_{(i,j) \in r_k} L_{ij} \le L$ and $d(r_{k^1}, r_{k^2}) \ge d^s, k^1, k^2 \in K$. In this definition, L is the maximum allowed depth of a given drilling path, given by the sum of all Euclidean distances L_{ij} between each pair of cells i and j.

In what follows, we introduce two MIP formulations for the aforementioned problem. The first one consists of a compact model, i.e., the number of variables and constraints can be described by a polynomial expression on the number of target cells. The second formulation is non-compact since it is based on an exponential number of constraints and must be solved by an iterative procedure to guarantee feasibility.

3.1 Compact Formulation

Given the above definitions, we define the following decision variables: (i) $x_{ijk}, i, j \in V, k \in K$, is a binary variable that equals 1 if path k visits target cells i and j in this specific order, and 0 otherwise; (ii) $y_{ik}, i \in V, k \in K$, is valued 1 if cell i is visited by path k, and 0 otherwise; (iii) $u_{ik} \in \mathbb{Z}^+$ representing the order in which cell i is visited within path k. The multi-directional drilling path optimisation problem can be defined as in Formulation (3a)–(3j), with $u_{0k} \coloneqq 0, k \in K$.

$$\max \sum_{i \in V'} \sum_{k \in K} p_i y_{ik} \tag{3a}$$

$$\text{s.t.} \sum_{j\in V'}\sum_{k\in K} x_{0jk} = \sum_{i\in V'}\sum_{k\in K} x_{i(n-1)k} = m \tag{3b}$$

$$\sum_{k\in K} y_{ik} \leq 1, \forall i \in V \tag{3c}$$

$$\sum_{i\in V\setminus\{n-1\}} x_{ijk} \geq y_{jk}, \forall j \in V \setminus \{0\}, k \in K \tag{3d}$$

$$\sum_{j\in V\setminus\{0\}} x_{ijk} \geq y_{ik}, \forall i \in V \setminus \{n-1\}, k \in K \tag{3e}$$

$$\sum_{i\in V\setminus\{n-1\}}\sum_{j\in V\setminus\{0\}} L_{ij}x_{ijk} \leq L, \forall k \in K \tag{3f}$$

$$x_{ijk} + x_{lmk} \leq 1, \forall (i,j,l,m) \in \bar{A} \tag{3g}$$

$$u_{jk} \geq u_{ik} + 1 - (n-1)(1 - x_{ijk}), \forall i,j \in V \setminus \{0\}, k \in K \tag{3h}$$

$$x_{ijk}, y_{ik} \in \{0,1\}, \forall i,j \in V, k \in K \tag{3i}$$

$$u_{ik} \in \mathbb{Z}^+. \tag{3j}$$

In this formulation, the objective function (3a) maximises the total prize collected by the designed paths among the visited MCC cells. Constraint (3b) makes sure that exactly m paths are generated. Constraints (3c) allow each cell to be visited at most once by any path, while Constraints (3d) and (3e) control which cells are visited or not. The maximum allowed measured length of each path is limited by Constraints (3f). Collisions are avoided by means of the $\mathcal{O}(n^4)$ constraints (3g). MTZ constraints (3h) avoid solutions containing subcycles [12]. Further information about such constraints is available in any literature on routing problems, e.g., [18].

The applicability of the formulation (3a)–(3j) extends to any arbitrary set of general parameters L, m, α and d^s, alongside cell-specific parameters p_i and L_{ij}, $i,j \in V$. The consideration of uncertainties related to these parameters is omitted in our analysis, as addressing such uncertainties would necessitate distinct optimisation methodologies, such as robust or stochastic optimisation [6, 14]. Such approaches are beyond the scope of the current paper. The technical team engaged in the exploration field is responsible for selecting suitable values for the above parameters.

Classical formulations for TOPs usually suffer from symmetries within their feasible region [1]. For the sake of completeness, we discuss the type of symmetries that most affect formulation (3a)–(3j) by using an example. Suppose an optimal solution for this problem consists of $m = 5$ paths visiting any subset of V, there are $m! = 5! = 120$ possible ways to re-index the paths while obtaining equivalent solutions.

Our formulation's search space can be reduced by incorporating two well-known symmetry-breaking constraints from the literature [13]. The first one (SCB1), or Constraints (4), enforces that path k must be assigned before path $k+1$. Therefore, a natural ordering of the paths is guaranteed. The second set of symmetry-breaking constraints (SCB2), or Constraints (5), also introduces an order between paths but employs a different mechanism. [13] notices that the total prize collected by each path is decreasing with the number of paths m. Therefore, by enforcing such ordering, paths are not exchangeable among themselves and equivalent re-indexed solutions can be discarded.

$$(SBC1) \quad y_{0k} \geq y_{0(k+1)}, \forall k \in K \setminus \{m-1\} \tag{4}$$

$$(SBC2) \quad \sum_{i\in V'} p_i y_{ik} - \sum_{i\in V'} p_i y_{i(k-1)} \geq 0, \forall k \in K \setminus \{0\} \tag{5}$$

From the point of view of a practitioner, formulation (3a)–(3j) presents some advantages, such as being relatively easy to understand and implement. In addition, a compact model allows the use of popular modelling languages and off-the-shelf optimisation software to solve real-world problems of moderated size. However, for our case study, the UNISIM-I reservoir model, such formulations may present some challenges. The main one is the need to preprocess the whole set $\bar{A}$ to implement constraints (3g). We recall that our MCC procedure resulted in $|V| := 101$ target cells that can be potentially visited in an optimal solution. Therefore, to fully preprocess set $\bar{A}$ the prohibitive computation of $104,060,401$ distances would be necessary, in the worst case. This is a non-trivial, time-consuming and computationally expensive task that must be avoided if one intends to solve larger real-world instances.

3.2 Non-compact Formulation and Branch-and-cut Algorithm

Let $\mathcal{R}$ be the set of all infeasible routes with respect to the safety distance and subcycle elimination constraints (i.e., (3g) and (3h), respectively). Also, let $A(r)$ denote the set of arcs in route $r \in \mathcal{R}$. Using this notation, we state the non-compact formulation as in (6).

$$\max \sum_{i \in V'} \sum_{k \in K} p_i y_{ik} \tag{6a}$$

$$\text{s.t. Constraints (3b)} - \text{(3f) and (3i)} - \text{(3j)} \tag{6b}$$

$$\sum_{(i,j) \in A(r)} x_{ijr} \leq |A(r)| - 1, \qquad \forall r \in \mathcal{R}. \tag{6c}$$

As in formulation (3), the objective function (6a) aims at maximising the overall collected prize. Constraints (3g) and (3h) were omitted because they are implicit from (6c).

In what follows, we describe a branch-and-cut (B&C) algorithm based on the combinatorial relaxation of formulation (6) regarding constraints (6c). In the proposed B&C, these constraints are separated for each incumbent candidate integer solution, with this process starting with checking existing subtours in each route of this solution. For each detected subtour, we generate a valid inequality (cut) using constraints (6c) and add it to the model (6). During the inspection, we store all the routes that compose this solution. The second step of our separation procedure checks for possible collisions among paths. That is, for any two paths r_{k^1} and r_{k^2} in the current integer solution, $k^1, k^2 \in K$, we check if $d(r_{k^1}, r_{k^2}) < d^s$. If such a collision is verified, we add a constraint of the type $x_{ij} + x_{lm} \leq 1$ for the corresponding arcs $(i,j) \in r_{k^1}$ and $(l,m) \in r_{k^2}$. This iterative procedure tends to avoid the computation of the whole set $\bar{A}$.

4 Computational Experiments

Both formulations were implemented through the `Pyomo` (v. 6.4.4) modelling language and solved by the software `CPLEX` (v. 12.7) in an Intel i7 CPU with 3.60GHz and 24GB of RAM running under Linux Mint 20.2 64bits. A time limit of 1h was set for each optimisation run and all instances were solved to optimality. Oil recovery simulations were carried out using CMG's IMEX software over a 40-year period.

In order to keep the presentation short, we omit computational results regarding the computational efficiency of the proposed B&C algorithm. However, our computational experiments demonstrated its superiority over the solution of the compact formulation.

4.1 Production Analysis

To analyse the technical and economic feasibility of the proposed methodology, we carried out tests of cumulative oil production (COP) and oil recovery factor (ORF). The ORF measures how much of the oil in place can be extracted using current technology. Both COP and ORF metrics are commonly used in the scientific literature and industry [8].

Using these metrics, we compared the results found by the approach proposed in this work for different problem parameters, such as the maximum measured length L and number of boreholes m. For example, $well1000$, $well2000$ and $well3000$ represent groups of four directional wells with $L = 1000\,m$, $2000\,m$ and $3000\,m$, respectively. These are also compared with the original vertical wells drilled in the Campos Basin [5] (denoted *Originals*), the wells proposed by [16] (denoted M1M2M3M4), and a single directional well presented in [7] (denoted $DP70$).

Our preliminary results show that $well2000$ almost doubled the ORF obtained by $DP70$. In general, the newly found wells perform better than other results found in the literature. Such results showcase the advantages of optimally planning multi-directional wells. The data from the experiments can be seen in figure 1 and reinforce that the approach presented is robust, efficient and promising for optimizing petroleum exploration, offering valuable insights to improve future research and industrial practice.

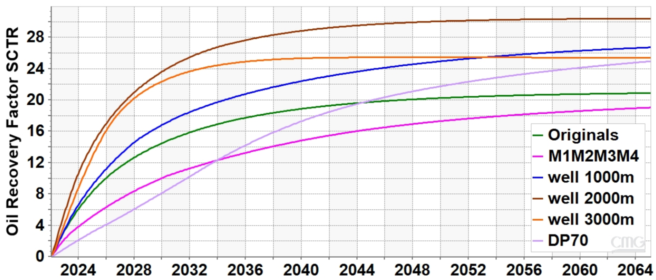

Figure 1: Oil recovery factor – ORF. Fonte: Autores.

5 Conclusions

This work presents an optimisation model for multi-directional petroleum exploration using MIP and flow simulations. We propose a compact formulation and a branch-and-cut algorithm for solving the problem. Preliminary results showed the computational efficiency of the proposed exact algorithm. A case study from Brazil's Campos Basin demonstrates the potential of optimized drilling paths to exceed the basin's historical recovery rates significantly. This novel approach promises significant advancements in directional drilling and the optimisation of offshore oil and gas production.

Acknowledgements

This work was partially supported by the CNPq (grant number 405654/2022-7) and CAPES (project number 88881.917250/2023-01).

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