Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, Vol. 3, N. 1, 2015.
Trabalho apresentado no XXXV CNMAC, Natal-RN, 2014.

Conjugate fuzzy QL-implications obtained by the OWA-operator*

Íbero C. K. Benítez **Renata H. S. Reiser** **Adenauer C. Yamin**
PPGC, CDTEC, UFPel - Pelotas, RS
E-mail: {ickbenitez, reiser, adenauer}@inf.ufpel.edu.br

Benjamín R. C. Bedregal
PPSC/PPgEEC - DIMAP - UFRN - Natal, RN
E-mail: bedregal@dimap.ufrn.br

ABSTRACT

Fuzzy logic is a powerful theory to make a decision which is usually irresolute for one thing or another, making it difficult to reach a final agreement. So, fuzzy connectives have been extensively studied in computer science and widely used in practical applications such as decision-making pattern recognition also including medical diagnosis, clustering analysis and image processing [3].

Fuzzy implications and aggregation functions are mainly study in this work in order to obtain new representative members in the class of fuzzy QL-operators by analysing related mathematical properties.

Thus, the aggregating fuzzy QL-subimplications are introduced. They are obtained by action of the OWA-operator performed over the family of the product triangular subnorms along with standard fuzzy negation and the probabilistic sum. As the main results, this family of QL-subimplications extend related QL-implications by preserving their corresponding properties. For that, let U be the unitary interval ($U = [0, 1]$).

Consider the subconorm $S_i\colon U^2 \to U$, $S_i(x, y) = 1 - \frac{1}{i}(1 - x - y + xy)$ for $i \geq 1$, the product t-norm $T\colon U^2 \to U$, $T(x, y) = xy$ and the standard fuzzy negation $N_S\colon U \to U$, $N_S(x) = 1 - x$.

An n-tuple of real numbers belonging to U^n can be aggregate to a single real number on U by an aggregation function, which is a non-decreasing operator satisfying the following boundary conditions:

$$A(0, 0, \ldots, 0) = 0 \text{ and } A(1, 1, \ldots, 1) = 1.$$

Let $\sigma\colon \mathbb{N}^n \to \mathbb{N}^n$ be a permutation function ordering the elements: $x_{\sigma(1)} \leq x_{\sigma(2)} \leq \ldots \leq x_{\sigma(n)}$. Let $w_1, w_2, \ldots, w_n$ be non negative weights ($w_i \geq 0$) such that their sum equals one ($\sum_{i=0}^{n} w_i = 1$). For all $\vec{x} \in U^n$, the n-ary aggregation function $A\colon U^n \to U$ called OWA-operator is given as:

$$A(\vec{x}) = \sum_{i=0}^{n} w_i x_{\sigma(i)}. \quad (1)$$

According with [4, 5], a function $I : U^2 \to U$ is a **fuzzy subimplication** if it satisfies the boundary conditions $I(1, 1) = I(0, 1) = I(0, 0) = 1$ together with the left antitonicity and right isotonicity. When a subimplication also verifies $I(1, 0) = 0$, it is called a **fuzzy implication** [1]. Additionally, QL-(sub)implication is a fuzzy (sub)implication defined, for all $x, y \in U$ by the following expression:

$$I_{S,N,T}(x, y) = S(N(x), T(x, y)), \quad (2)$$

when $T(S)$ is a t-(co)norm and N is a strong fuzzy negation.

*This work is supported by the Brazilian funding agencies CAPES and FAPERGS (Ed. PqG 06/2010, under the process number 11/1520-1).

DOI: 10.5540/03.2015.003.01.0096

Proposition 1. *Let $\rho : U \to U$ be an automorphism [2]. The function $J_i : U^2 \to U$ and its conjugate fucntion $J_i^\rho : U^2 \to U$, given in Eq.(3) and Eq.(4) respectively, are both fuzzy QL-subimplications:*

$$J_i(x,y) = S_i(N_S(x), T_P(x,y)) = 1 - \frac{1}{i}(x - x^2 y), \quad (3)$$

$$J_i^\rho(x,y) = S_i^\rho(N_S^\rho(x), T_P^\rho(x,y)) = \rho^{-1}\left(1 - \frac{\rho(x)}{i}(1 - \rho(x)\rho(y))\right), \ \forall x,y \in U. \quad (4)$$

Such family of fuzzy QL-subimplications is referred as $\mathcal{J}$.

Proposition 2. *The QL-subimplication $J_i, J_i^\rho \in \mathcal{J}$ verify the properties:*

I1 : *If $S(N(x),x) = 1$ then $I(x,1) \leq 1$, for all $x \in U$;*

I2a : *If $x_1 \geq x_2$ then $I(x_1,0) \leq I(x_2,0)$, for all $x_1, x_2 \in U$.*

I2b : *If $y_1 \geq y_2$ then $I(1,y_1) \leq I(1,y_2)$, for all $y_1, y_2 \in U$.*

In [6], an k-ary function $\mathcal{F}_A : U^k \to U$ is called as $(A,\mathcal{F})$**-operator** and given by:

$$\mathcal{F}_A(x_1,\ldots,x_k) = A(F_1(x_1,\ldots,x_k),\ldots,F_n(x_1,\ldots,x_k)), \ \forall x_1,\ldots,x_k \in U. \quad (5)$$

Proposition 3. *Let $\mathcal{T}_P = \{S_i(x,y) = 1 - \frac{1}{i}(1 - x - y + xy) : i \geq 1\}$ be a family of t-subnorms. The function $\mathcal{T}_{OWA} : U^2 \to U$ is a t-subconorm given by Eq.(6) in the following:*

$$\mathcal{S}_{OWA}(x,y) = \sum_{i=0}^{n} w_i S_{\sigma(i)}(x,y), \forall x,y \in U. \quad (6)$$

As the main result, we present the subclass of fuzzy QL-subimplication represented by a t-norm T_P, the standard negation N_S together with a t-subconorm $\mathcal{S}_P$, which is obtained by aggregating n fuzzy t-subconorms of the family $\mathcal{S}_P$.

Theorem 1. *For all $x,y \in U$, the function $\mathcal{J}_{OWA} : U^2 \to U$ is a QL-subimplication, given by*

$$\mathcal{J}_{OWA}(x,y) = \mathcal{S}_{OWA}(N_S(x), T_P(x,y)). \quad (7)$$

Proposition 4. *The QL-subimplication $\mathcal{J}_{OWA}$ verify* **Ik** *for* $\mathbf{k} \in \{1, 2a, 2b\}$.

Concluding, by Prop. 4, the operator $\mathcal{J}_{OWA}$ preserves the main properties of QL-subimplications. Further work considers the interrelations between this class of subimplications and their possible conjugate functions. Another interesting issue is related to dual constructions of QL-subimplications.

Keywords: *OWA-operator, fuzzy QL-(sub)implications; fuzzy (sub)implications.*

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DOI: 10.5540/03.2015.003.01.0096