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Discrete Robust Features on Piecewise Linear Surfaces

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Surfaces in $\mathbb{R}^3$ have features that capture key aspects of their local differential geometry, called robust features, which can be followed when the surface is deformed, that is, the features persist on the deformed surfaces. Amongst these robust features, we have the parabolic and ridge curves. Given a smooth surface M in $\mathbb{R}^3$, the parabolic set of M is the zero set of its Gaussian curvature. The ridge is the set of points on M where a principal curvature is extremal along its own lines of principal curvature (colored red for one principal curvature and blue for the other).

We consider here discrete robust features of discrete surfaces S in $\mathbb{R}^3$. Usually, this is done by discretizing the robust features of a smooth approximation of the discrete surface. In our study, we work directly with discrete surfaces using tools from Discrete Differential Geometry (see for example [1, 2, 5]). Our surfaces are piecewise linear surface meshes.

The discrete Gaussian curvature $K(v)$ at a vertex v on a discrete surface S is defined as

$$K(v) = (2\pi - \theta(v))\,/A(v), \tag{1}$$

where $\theta(v)$ is the angle defect and $A(v)$ is the area of the barycentric region of the vertex v.

Let vv_j be an edge of S, and f_1 and f_2 the adjacent faces that have vv_j as a common edge. The angle between the faces is called the dihedral angle and is defined by

$$\varphi(vv_j) := \text{atan2}\left(\frac{v_j - v}{\|v_j - v\|} \cdot n_{vv_jv_k} \times n_{v_jvv_l}, n_{vv_jv_k} \cdot n_{v_jvv_l}\right), \tag{2}$$

where n_{abc} is a unit normal vector to the face with vertices a, b, c. Let v be a vertex of S, and v_j, $j = 1, \ldots, l$, the vertices of the star of v. The discrete mean curvature of S at v is defined as

$$H(v) = \frac{1}{4A(v)} \cdot \sum_j \|v - v_j\| \varphi_{vv_j}. \tag{3}$$

The discrete principal curvatures $\kappa_1(v)$ and $\kappa_2(v)$ are defined as the solutions of the quadratic equation $\kappa^2 - 2H(v)\kappa + K(v) = 0$. The associate discrete principal direction $d_1(v)$ and $d_2(v)$ are the eigenvectors of the 2×2 matrix B (analogous to the matrix of the shape operator) which satisfies $d_j^T B d_j = k_j^N$, for all v_j in the star of v, where d_j is the unit direction in the tangent plane of the edge vv_j and $k_j^N = 2(v - v_j) \cdot N(v)/\|v - v_j\|^2$, with $N(v)$ the normal vector at v (see [5]). The discrete ridge is the zero set of the directional derivative of κ_i in the direction d_i, $i = 1, 2$.

We developed in [4] a code for drawing discrete robust features on discrete surfaces. As a result, we obtained new discrete ridge curves on the discretization of the graph of the smooth function $z = h(x, y)$ considered in [6] that were missed in that study; compare the two figures in Figure 1.

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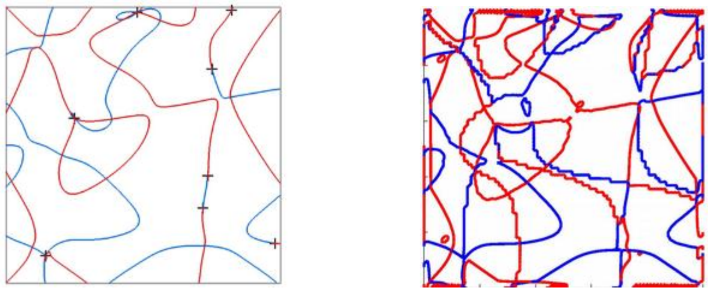
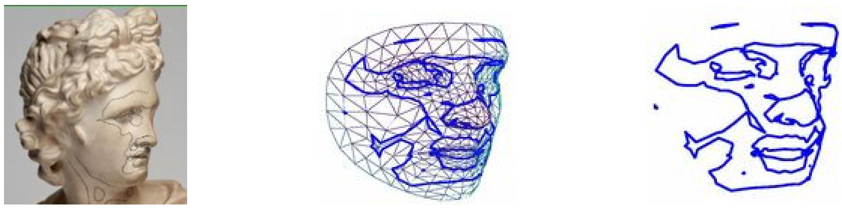


(a) (b)

Figure 1: Ridges curves of the graph of $h(u, v)$, left from [6], and discrete ridge curves of the discretization of that surface, right. Fontes: left from [6], right from the authors.

Felix Klein drew parabolic curves on the bust of Apollo Belvedere. As a comparison, we applied our code to the mesh of the bust of Apollo and found its discrete parabolic curves; see Figure 2.



(a) (b) (c)

Figure 2: Parabolic curves on the bust of Apollo Belvedere (left figure), and their discrete analogue (middle and right figures). Fontes: left from [3], middle and right from the authors.

Acknowledgments

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References

[1] K. Crane. **Discrete Differential Geometry - CMU 15-458/858**. Aula 16. URL: `https://www.youtube.com/watch?v=NlU1m-OfumE`.

[2] K. Crane. **Discrete Differential Geometry - CMU 15-458/858**. Aula 17. URL: `https://www.youtube.com/watch?v=sokeN5VxBB8`.

[3] F. Klein. **Apollo Belvedere with parabolic curves**. 1910. URL: `https://collections.ed.ac.uk/coimbra/record/97358`.

[4] D. A. Marques. "Características robustas discretas em superfícies lineares por partes". In preparation. PhD thesis. University of São Paulo.

[5] M. Meyer et al. "Discrete differential-geometry operators for triangulated 2-manifolds". In: **Visualization and mathematics III**. Springer, 2003, pp. 35–57.

[6] S. Musuvathy et al. "Principal curvature ridges and geometrically salient regions of parametric B-spline surfaces". In: **Computer-Aided Design** 43 (July 2011), pp. 756–770. DOI: `10.1016/j.cad.2010.09.013`.