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Elementary Zip-Cellular Automaton

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Cellular Automata (CA) have turned out to be a very fruitful approach to solving many scientific problems, providing an efficient way to model and simulate specific phenomena for which more traditional computational techniques are hardly applicable. A cellular automaton is a discrete dynamical system that evolves a grid of cells, updated according to some local and global rules. The evolution of the state of each cell as a function of the state of the neighboring cells takes place using the local rule, and the global evolution rule governs the overall behavior of the grids. Around the 1960s they were studied as a dynamical system and their connection with the mathematical field of symbolic dynamics was established for the first time [1], [2]. In this work, we use some extended symbolic dynamics to modify such construction into a new generation of CA, the so-called Zip-Cellular Automaton, which includes the interaction of two local rules. From a dynamical system point of view, this new generation of CA is any finite-to-1 continuous map of a zip shift space that commutes with zip shift maps and have mostly the same dynamical properties of a classical CA.

Consider two finite sets of alphabets $\mathcal{Z} = \{a_1, \ldots, a_k\}$ and $\mathcal{S} = \{0, \ldots, m-1\}$. Let $\Sigma_{\mathcal{S}} \subseteq \{(x_i)_{i\in\mathbb{Z}},\, x_i \in \mathcal{S}\}$ be a shift space (for details on shift space see for instance [**3**]).

Definition 1. *For some fixed $N = m + n + 1 \in \mathbb{N}$, let $\tau_n : B_n^{\mathcal{S}} \longrightarrow \mathcal{Z}$ be a surjective transition map, which is not necessarily invertible, with B_n^S being the set of all admissible words of length n. Let $Y = \{y = (y_i)_{i\in\mathbb{Z}},\, y_i \in \mathcal{S}\}$ be a two-sided full shift space with its associated shift map σ. For any point $y \in Y$, we correspond a point $x = (x_i)_{i\in\mathbb{Z}} = (\ldots, x_{-2}, x_{-1};\, x_0,\, x_1,\, x_2, \ldots)$ such that*

$$x_i = \begin{cases} y_i \in \mathcal{S} & i \geq 0 \\ \tau_n(y_{[i-n,i+m]}) \in \mathcal{Z} & i < 0. \end{cases} \tag{1}$$

Then consider the following space: $\Sigma_{\mathcal{Z},\mathcal{S}} := \{x = (x_i)_{i\in\mathbb{Z}} \ : x_i \text{ satisfies} (1)\}$. The space $\Sigma_{\mathcal{Z},\mathcal{S}}$ is so called the "full zip-shift space".

Definition 2. *The zip-shift map $\sigma_\tau : \Sigma_{\mathcal{Z},\mathcal{S}} \to \Sigma_{\mathcal{Z},\mathcal{S}}$ is defined as follows. For every $i \in \mathbb{Z}$,*

$$(\sigma_\tau(x))_i = \begin{cases} \tau_n(x_0 \ldots x_{n-1}) & i = -1 \\ x_{i+1} & \text{otherwise.} \end{cases}$$

Note that the map σ_τ is well-defined.

It is well-known that the shift map is homeomorphism. In the case of zip-shift map we have the following Theorem.

Theorem 1. *The zip shift map is a local homeomorphism.*

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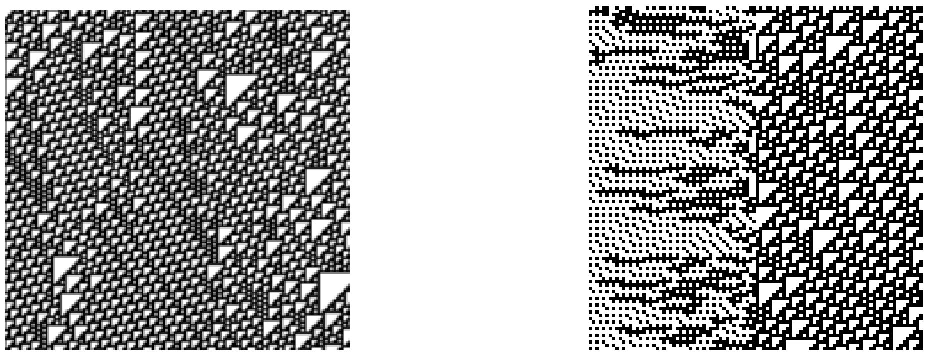
The configuration c of an uni-dimentional **Elementary Cellular Automaton** (ECA) with state set $\mathcal{S} = \{0, 1\}$ in general is an application $c : \mathbb{Z} \to \mathcal{S}$ that specifies the state of each cell in a grid of cells. The set of all possible configurations of a grid is represented by $\mathcal{C}$. The **local rule** is an application $\mathcal{R} : \mathcal{S}^{2n+1} \to \mathcal{S}$ where n represents the neighborhood radius. The **global transition function** $G : \mathcal{C} \to \mathcal{C}$, where $G(c) = e$ is a new setting in $\mathcal{C}$. [2]

Definition 3 (**zip-shift based sliding block codes**). *Let $\mathcal{Z}, \mathcal{S}$ and $Q, P = \{0, 1\}$ be two collections of finite alphabet sets (state spaces). Assume that $(\Sigma_{\mathcal{Z},\mathcal{S}}, \sigma_\tau)$ and $(\Sigma_{Q,P}, \sigma_\kappa)$ represent full zip-shift spaces. Set $N = m + n + 1$ and $M = m' + n' + 1$, where m, m' are the memories and n, n' are the anticipations. A continuous map $R : \Sigma_{\mathcal{Z},\mathcal{S}} \to \Sigma_{Q,P}$ is a zip-shift-sliding block code if*

- *there exist a local rule $R = R_2 \circ R_1$, with code maps $\phi_{R_1} : B_N(\Sigma_{\mathcal{Z},\mathcal{S}}) \to P^{\mathbb{Z}}$ and $\phi_{R_2} : B_M(P^{\mathbb{Z}}) \to \Sigma_{Q,P}$;*
- *R commutes with zip-shift maps.*

Moreover, we call R an Elementary Zip-Celular Automaton (Zip-CA) when $Z = Q = S = P$.

In what follows we give an examples of an Elementary cellular which represents the famous Wolfram elementary rules R150 and R110 [3] and an Example of an Elementary zip-cellular automaton that represents rule R110 in interaction with rule R75 [3].



(a) (b)

Figura 1: (a) R110. (b) R110 interaction with R75 zip-cellular automata. Source: authors

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