Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, Vol. 3, N. 1, 2015.
Trabalho apresentado no XXXV CNMAC, Natal-RN, 2014.

Computer assisted research: Using SYM for the symmetry analysis of Differential Equations

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Abstract

SYM is a symbolic package — developed in the computer algebra system Mathematica — for the symmetry analysis of Differential Equations (DE) [5, 8]. And since its initial concept on 2002 it is in constant development by the author.

The purpose of the package is to function as a symbolic toolbox of commands for the symmetry analysis, and manipulation of those symmetries, in the study of DE. It consists of fully automated commands and *low level* ones in order to cover a wide range of tasks: from automatically retrieve the classical Lie point symmetries of a DE to interactively obtain the approximate symmetries or constructing conservation laws using the previously found symmetries of a DE to name a few. Since its first presentation back at 2005, [3], many new commands were added augmenting its functionality. Currently, SYM consists of more than a hundred commands divided loosely into there categories: Symmetry analysis, Lie Algebra analysis and Auxiliary commands.

In the present poster an exposition of the advancements of the package since 2005 will be given. Apart from a general review of its structure and its basic functionality the following aspects of the package will be illustrated:

- Analysis of the algebraic properties of the symmetries of a DE, solvability, nullity, derived series, radical, quotient space, change of variables Levi decomposition etc..
- Equivalence transformations: group classification of classes of DE containing arbitrary parameters and functions [8, 6, 9]. We show how to obtain the simplest representative of a class of DE simplifying hence its classification.
- Approximate symmetries: symmetry analysis of perturbed DEs from the point of view of the perturbation theory [1, 4, 2]. We consider three different ways and obtain perturbed closed solutions for each one of them.
- Conservation laws: construction of conservation laws using the symmetries of a DE, even when the DE does not have a variational form or is of even order [7]. Classification of a class of DE by the self–adjointness property and non–trivial conservation laws.

Keywords: *computer assisted research, symmetries of DE, conservation laws*

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DOI: 10.5540/03.2015.003.01.0107