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Zero-Lag Synchronization of Bidirectionally Coupled Chaotic Oscillators with Delay

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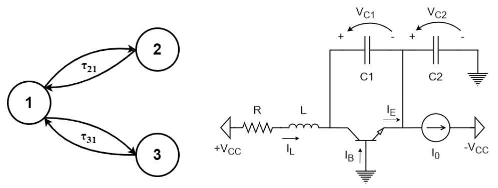
Synchronization is an important phenomenon in fields of science such as physics, biology and engineering, whereby the mutual coupling of oscillators gives rise to collective dynamic behavior. This behavior is key to understanding complex neural networks (NN). For example, studies suggest that, underlying cognitive activities, synchronization of distant cortical regions occurs in the brain [1]. Synchronization may occur without delay, overcoming the physical distance between the synchronized regions, and is then called zero-lag synchronization. For a comprehensive analysis, different types of coupling, mechanisms and complex architectures of NN need to be considered. Figure 1(a) presents a basic network topology that consists of two oscillators connected through bidirectional coupling to a central oscillator with delays τ_{21} and τ_{31}. Such a system has already been studied using lasers [2]. Here, we investigate the behavior of this configuration using electronic Colpitts oscillators (Figure 1(b)), whose dynamics is easily controlled through the oscillator current, allowing operation in different regimes, particularly chaotic ones.



(a) (b)

Figure 1: a) Network with three oscillators. b) Colpitts in common base configuration. Source: authors.

The dynamics of oscillator i ($i = 1, 2, 3$) are described by normalized state equations [3],

$$\begin{cases} \dot{x}_i = \frac{g^*}{Q(1-K)}[-n(y_i) + z_i] \\ \dot{y}_i = \frac{g^*}{QK} z_i \\ \dot{z}_i = -\frac{QK(1-K)}{g^*}[x_i + y_i] - \frac{1}{Q} z_i, \end{cases} \quad (1)$$

$$n(y_i) = \begin{cases} -y_i, & y_i \leqslant 1 \\ -1, & y_i > 1, \end{cases} \quad (2)$$

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where z_i is the normalized current on inductor L, x_i and y_i are, respectively, the normalized voltage on capacitors C_1 and C_2, Q is the RL tank quality factor ($Q = \frac{\omega_0 L}{R}$), K is the capacitive voltage divider ($K = \frac{C_2}{C_1+C_2}$), $g^* = \frac{\alpha_F I_0 L}{V_T R(C_1+C_2)}$, α_F is a gain of the transistor current and V_T is the thermal voltage. We consider ideal conditions ($\alpha_F = 1$) [3]. The resistive coupling between the oscillators can be expressed by $\epsilon(y_i - y_j)$. We set the coupling strength factor $\epsilon = 1$. We use the Bogacki-Shampine method for numerical integration of delayed differential equations [4] and find that synchronization with zero delay can be achieved numerically. Zero-lag synchronization was also achieved experimentally with two Colpitts oscillators operating in a chaotic regime and connected through a third oscillator, with $C_1 = 10nF$, $C_2 = 3.3nF$, $L = 220uH$ and a delay produced by an electronic module. Figure 2 presents time series of the three oscillators and the corresponding cross-correlation signal (Equation 3) for each pair of oscillators:

$$C_{ij} = \frac{\langle x_i(t)x_j(t+\Delta t)\rangle}{\sqrt{\langle x_i^2(t)\rangle\langle x_j^2(t)\rangle}} \tag{3}$$

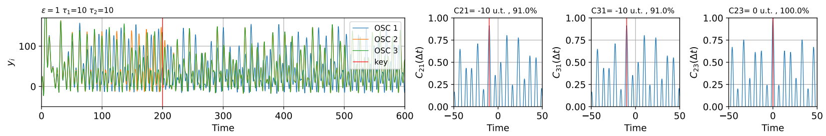

Figure 2: Time series of three Colpitts oscillators coupled with equal delays and cross-correlation between the three oscillators for the topology of Figure 1(a). Source: authors.

Zero-lag synchronization between the two external oscillators is evidenced by the cross-correlation coefficient C_{23} [5][2]. Synchronization also occurs between the central oscillator and the external ones, although delayed (see C_{21} and C_{31}). The synchronization pattern can be controlled through the delay between the oscillators. For example, for coupling delays $\tau_{21} = 2\tau_{31}$, both oscillators 1 and 2 synchronize with zero-lag, lagging behind oscillator 3. We numerically show that zero-lag behavior also occurs in networks of more than three Colpitts oscillators, with the emergence of groups of synchronized oscillators (sublattice synchronization), depending on the parity of the number of oscillators. As a development, the zero-lag behavior discussed here may be explored in more complex neurons motifs and is a promising technique for applications e.g in communication and neural systems.

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