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Lyapunov Exponents for a Family of Dissipative Two-dimensional Mappings

Juliano A. Oliveira[1]
UNESP, São João da Boa Vista, SP
Fábio H. Costa[2] Mayla A. M. Almeida[3] Edson D. Leonel[4]
IGCE/UNESP, Rio Claro, SP

In this work we consider a family of dissipative two-dimensional mappings and the Lyapunov exponents to characterize the chaos [1, 2]. The mapping is defined as [2]:

$$T : \begin{cases} I_{n+1} = |\delta I_n - (1+\delta)\epsilon sin(2\pi\theta_n)|, \\ \theta_{n+1} = \theta_n + I_{n+1}^{\gamma} \quad mod(1) \end{cases} \tag{1}$$

where θ and I are angle-action variables, ϵ controls the nonlinearity, δ controls the dissipation magnitude and γ is a dynamical exponent that recovers several dynamical systems known in the literature. For $\delta = 1$ the conservative case is recovered such as area-preserving in the phase space is observed. Fig. 1 shows the phase space for different initial conditions uniformly distributed in the window $I_0 \in [0.001, 1]$ and $\theta_0 \in [0.1, 1]$ iterated up to 10^4 times. We can observe chaotic attractors for different values of ϵ as labeled in the figure.

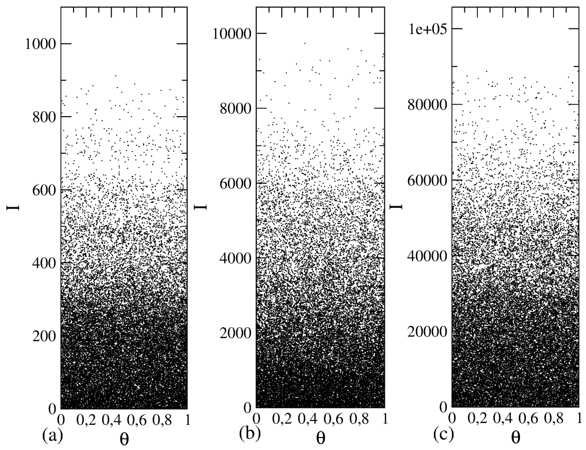

Figure 1: Phase space for $\gamma = 1$, $\delta = 0.999$ and (a) $\epsilon = 10$, (b) $\epsilon = 10^2$ e (c) $\epsilon = 10^3$. Source: The authors.

Our main goal of investigation is to characterize the chaotic attractors using the Lyapunov exponents defined as [1]:

$$\lambda_j = \lim_{n\to\infty} \frac{1}{n} ln|\Lambda_j^{(n)}|, \quad j = 1, 2, ..., \tag{2}$$

[1]juliano.antonio@unesp.br
[2]fabio.costa@unesp.br
[3]mayla.almeida@unesp.br
[4]edson.leonel@unesp.br

where $\Lambda_j^{(n)}$ are the eigenvalues of the matrix $M = \prod_{i=1}^{n} J_i(I_i, \theta_i)$ such that J_i is the Jacobian matrix of the mapping evaluated along the orbit. Fig. 2 shows the Lyapunov exponents for Eq. (1) using $\epsilon = 10$, $\delta = 0.999$ $\gamma = 1$ and five initial conditions (as labeled in the figure). Fig. 2(a) shows the positive Lyapunov exponents λ_1. The average over the different orbits yields $\bar{\lambda_1} = 4.13988(5)$. Fig. 2(b) shows the negative Lyapunov exponent λ_2, with an average value of $\bar{\lambda_2} = -4.14088(5)$. Fig. 2(c) provides $\bar{\lambda_1} + \bar{\lambda_2} \approx -10^{-3}$, which is of the same magnitude of $(1 - \delta)$. In our investigations

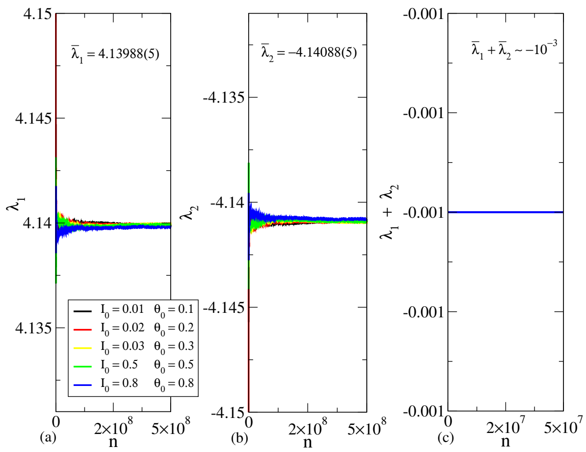

Figure 2: Plots of λ versus n for Lypaunov exponent (a) positive, (b) negative and (c) $\lambda_1 + \lambda_2 \approx -10^{-3}$. Source: The authors.

considered different values of the control parameter and noted that keeping ϵ fixed and using different values of δ a logarithmic increase for the average negative Lyapunov exponent was observed while the average of the positive Lyapunov exponent decreases logarithmically. For the case of δ fixed and varying ϵ a logarithmic decrease in the average of the negative Lyapunov exponent was observed while the average of the positive Lyapunov exponent increases logarithmically. Using ϵ and δ fixeds and different values of γ the average of the positive Lyapunov exponent decreases linearly while the average of the negative Lyapunov exponent increases linearly. Such results are important to understand the organization of the parameter space and several dynamical features in the system such as tangent, period-doubling, pitchfork and cusp bifurcations. Furthermore they can describe a complex set of streets with the same periodicity and the period-adding of spring and saddle-areas.

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