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A Five-piece Pythagorean Dissection of Hexagons

Jéssica F. Alcântara[1]
E.E.F.T.I. Dona Odorina Castelo Branco Sampaio, Juazeiro do Norte, CE
Valdinês L. S. Júnior[2], Erica B. Batista[3]
UFCA, Juazeiro do Norte, CE
Lucas V. Meireles[4]
IF Goiano, Posse, GO

The integration of Pythagorean dissections into the teaching of mathematics offers a fascinating avenue for students to explore geometric problem-solving techniques. This approach provides a fresh perspective on problem-solving, demonstrating the versatility and creativity inherent in Pythagorean dissections.

A Pythagorean dissection is a fascinating geometric method to illustrate the Pythagorean theorem. It involves dissecting two polygons into smaller pieces and rearranging them to form a larger polygon. There are many types of Pythagorean dissections, each with its own unique properties and applications. For instance, in 1973, James Schmerl presented in [2] a five-piece dissection of hexagons for the relation $3^2 + 4^2 = 5^2$. In Schmerl's dissection, the four-hexagon was left uncut, as shown in Figure 1.

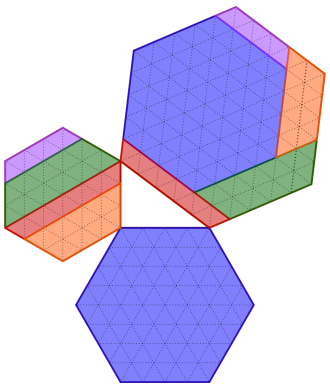

Figure 1: James Schmerl's five-piece dissection.

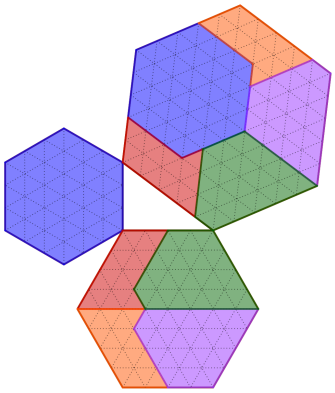

Figure 2: Greg Frederickson's five-piece dissection.

Similarly to Schmerl's dissection, Greg Frederickson, [1, page 98], proposed as a challenge the construction of a dissection for the relation $3^2 + 4^2 = 5^2$, but unlike Schmerl's dissection, in this problem, the three-hexagon should remain uncut. As a solution, see [1, page 282], the dissection given in Figure 2 was presented by Frederickson.

[1]jessicaferreirahj@gmail.com
[2]valdines.leite@ufca.edu.br
[3]erica.batista@ufca.edu.br
[4]autor3@email

As a response to Frederickson's challenge, we are pleased to present another five-piece dissection of hexagons in which a three-hexagon remains uncut, as proposed in the problem. The dissection is illustrated in Figure 3.

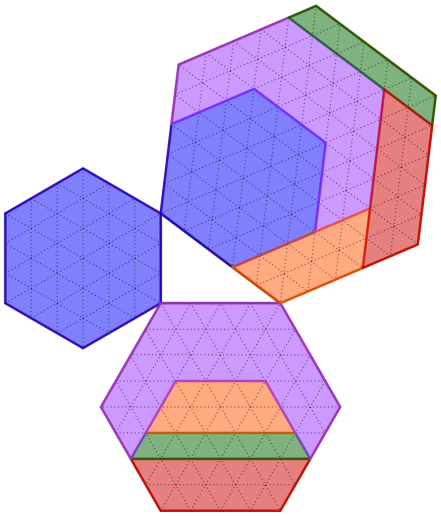

Figure 3: Proposed new five-piece dissection of hexagons.

In summary, the integration of Pythagorean dissections into mathematics education offers an intriguing pathway for students to delve into geometric problem-solving techniques. These dissections provide a fresh viewpoint, demonstrating the adaptability and ingenuity inherent in the Pythagorean theorem. The various types of Pythagorean dissections, exemplified by the works of James Schmerl and Greg Frederickson, showcase unique properties and applications, enriching the exploration of geometric concepts. The introduction of a new five-piece dissection of hexagons, addressing Frederickson's challenge while preserving the integrity of the three-hexagon, contributes to the pool of solutions in this area. This contribution underscores the ongoing conversation surrounding Pythagorean dissections and aims to stimulate further inquiry and research in the field of geometric problem-solving.

References

[1] G. N. Frederickson. **Dissections: plane and fancy**. Cambridge University Press, 1997.

[2] J. Schmerl. "Problem #240: A Pythagorean dissection," in: **Journal of Recreational Mathematics** 6 (1973), pp. 315–316.