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The SIR Metapopulation Model in a Temporal Commuting Network

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Network science serves as a powerful tool for analyzing mobility data, especially within the context of epidemiology. The COVID-19 pandemic highlighted the significance of human mobility patterns in disease transmission [3]. Researchers thus model and analyze mobility data by leveraging the formalism of complex networks, representing areas as nodes and movements between origins and destinations as edges. Moreover, mathematical models are instrumental in grasping disease transmission dynamics, including epidemics spanning large geographical areas, where long-distance travel is a significant factor. Real-world studies require coupling mobility data with host movement models like Eulerian and Lagrangian models [1], commonly used for movement, each with a distinct representation.

We present an adaptation of a SIR model following the Eulerian approach for inflow and outflow rates and use real temporal mobility networks as a case study. We leverage the Baidu Mobility Data [4], a dataset containing commuting patterns in China during the critical period of the COVID-19 pandemic, providing daily inflow and outflow rates of people between 303 Chinese cities over 60 days from January 1 to February 29, 2020. Previous research [2] conducted preprocessing procedures, resulting in the matrices w_{ij}^{in} and w_{ij}^{out} with the shape 303 x 303 that categorize mobility into inflows and outflows by province and city.

Regarding the Baidu Mobility Data, distinct patterns emerge when comparing daily matrices using Frobenius norm. Days 1 to 25 show close alignment between inflow and outflow networks, coinciding with major events like the Spring Festival (January 1), Wuhan Travel Ban (January 23), and Lunar New Year start (January 25). However, from day 26 to 60, a noticeable shift occurs, with increased disparity between mobility networks, coinciding with the Wuhan Travel Ban implementation (January 23) and evolving societal responses to COVID-19 concerns.

We employ the Flux (Eulerian) [1] model to simulate human mobility dynamics, followed by applying the SIR flux model to simulate disease transmission dynamics on top of mobility networks. The Flux model we use is given by:

$$\dot{N}_i = -\sum_{j=1}^{K} w_{ij}^{out} N_i + \sum_{j=1}^{K} w_{ij}^{in} N_j \tag{1}$$

in which K denotes the total number of subpopulations. $\dot{N}_i$ represents the rate of change of the number of hosts currently located at site i over time, $-\sum_{j=1}^{K} w_{ij}^{out} N_i$ describes the outflow of hosts from site i to other sites, where w_{ij}^{out} represents the proportion of hosts moving from site j to site i. Conversely, the term $\sum_{j=1}^{K} w_{ij}^{in} N_j$ gives the inflow of hosts from other sites to site i.

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This equation assumes a homogeneous mixing scenario, where hosts from different subpopulations come into contact with each other if they travel to occupy the same location. Incorporating the Eulerian movement model into the ordinary SIR compartmental model yields the following set of $3K$ equations:

$$\begin{cases} \dot{S}_i = -\beta_i \frac{S_i I_i}{N_i} - \sum_{j=1}^{K} w_{ij}^{out} S_i + \sum_{j=1}^{K} w_{ij}^{in} S_j \\ \dot{I}_i = \beta_i \frac{S_i I_i}{N_i} - \gamma I_i - \sum_{j=1}^{K} w_{ij}^{out} I_i + \sum_{j=1}^{K} w_{ij}^{in} I_j \\ \dot{R}_i = \gamma I_i - \sum_{j=1}^{K} w_{ij}^{out} R_i + \sum_{j=1}^{K} w_{ij}^{in} R_j \end{cases} \tag{2}$$

in which $-\sum_{j=1}^{K} w_{ij}^{out} S_i$, $-\sum_{j=1}^{K} w_{ij}^{out} I_i$, and $-\sum_{j=1}^{K} w_{ij}^{out} R_i$ represent the outflow of susceptible, infected, and recovered individuals from site i to other sites, respectively. Terms with the "in" superscript depict inflows.

We simulated the Eulerian SIR model with 5,000 hosts per node, including 4,900 susceptible individuals and 100 infected individuals. Transmission and recovery parameters were set to $\beta_i = 0.9$ and $\gamma = 0.2$, respectively. Figure 1 depicts the trajectories of compartments S, I, and R for one node.

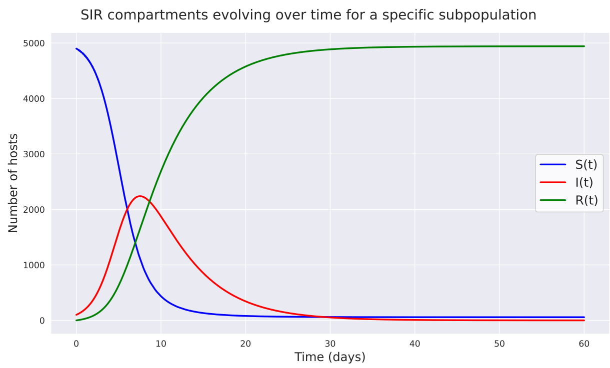

Figure 1: Eulerian SIR model compartments for a specific subpopulation. Source: the authors.

Our ongoing research explores the complex relationship between temporal resolution in mobility networks and epidemic dynamics. As we progress, our objectives include quantifying differences between simulated dynamics in temporal networks and their static version, while also investigating the network's tolerance to changes in temporal resolution compared to its more refined version.

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