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Sensitivity Analysis of a Heat Transfer Model for a Cold-Beverage Cup System

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The temperature sensitivity of many goods is essential for enhancing the quality and consumer experience in several industries, particularly in beverages. From the brewing of teas and the fermentation of wines to the preservation of freshness in fruit juices, maintaining the optimal temperature is critical, therefore, any beverage container should maintain its temperature as long as possible [5]. Mathematical modeling helps to achieve this by providing an overview of the heat transfer occurring within the beverage and its container. Afterward, the model should be assessed to determine the adjustment of the parameters to improve the thermal performance of the container [4].

Sensitivity analysis is a fundamental tool for understanding any model's behavior, structure, and response to changes in the input parameters. It can assess the impact of each input and its combinations on the variability observed in the model's output [2, 3]. This work aims to assess the parameters of a cold-beverage cup model to minimize heat gain using sensitivity analysis. The mathematical model considered for the heat transfer through the beverage contained in an open-top-cylindrical cup of radius R, height H, and thickness ϵ, at an initial temperature T_0 when exposed to quiescent air at room temperature T_∞, is given as follows,

$$\begin{cases} \dfrac{\partial(\rho T)}{\partial t} = \dfrac{\partial}{\partial x}\left(\dfrac{k}{C_p}\dfrac{\partial T}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{k}{C_p}\dfrac{\partial T}{\partial y}\right) \\ q''_{top} = 0.52\, Ra_L^{1/5}\dfrac{k_{air}}{L}(T_\infty - T_{s,top}) + \sigma(T_\infty^4 - T_{s,top}^4), \\ q''_{side} = \left\{\dfrac{1}{0.943}\left[\dfrac{\rho_{liq.sat}\, g\,(\rho_{liq.sat}-\rho_v)h'_{fg}H^3}{\mu_{liq.sat}\,k_{liq.sat}(T_{liq.sat}-T_{s,side_{outer}})}\right]^{-1/4}\dfrac{H}{k_{liq.sat}} + \dfrac{\epsilon}{k_{cup}}\right\}^{-1}(T_\infty - T_{s,side_{inner}}), \\ q''_{bottom} = 0, \\ q''_{simmetry} = 0. \end{cases} \tag{1}$$

where $L = R/2$ is the characteristic length, Ra is the Rayleigh number, g is the standard acceleration of gravity, σ is the Stephan-Boltzmann coefficient, and T_s is the surface temperature. k, μ, ρ, and C_p are the thermal conductivity, dynamic viscosity, density, and isobaric-specific heat of the beverage, respectively. Also, q'' is heat flux. Additionally, $h'_{fg} = h_{fg} + 0.68\, C_{p,\, liq.sat}(T_{liq.sat} - T_{s,side_{outer}})$, the subscript $liq.sat$ means saturated water, the subscript v means vapor, and h_{fg} is the latent heat of vaporization. All properties of saturated water should be evaluated at film temperature $T_f = 0.5(T_{liq.sat} + T_{s,side_{outer}})$. For ρ_v and h_{fg} should be estimated at $T_{liq.sat}$. All correlations considered are from [1]. The model accounts for heat transfer via natural convection and radiation on the beverage's free surface, film condensation along the cup's sides, and conduction

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within the cup. The bottom surface is insulated, and these processes occur symmetrically around the cup. The solution of equation (1) will return the temperature distribution T for a cross-section, with spatial dimensions x and y, due to symmetry, and it is obtained through a code implementation using the finite volume method.

For this particular model, the finite difference method is the selected sensitivity analysis technique. The method assigns a base case $\boldsymbol{\chi}^0$ and a sensitivity case $\boldsymbol{\chi}^+$ to the model inputs. The corresponding vector of the input changes as follows,

$$\boldsymbol{\Delta\chi}^+ = \boldsymbol{\chi}^+ - \boldsymbol{\chi}^0 = \langle \Delta\chi_1^+, \Delta\chi_2^+, ..., \Delta\chi_n^+ \rangle \tag{2}$$

Then, the model is solved by varying one input at a time from the base case to the sensitivity case. The sensitivity measures are given by,

$$\Delta\psi_i^+ = g(\chi_i + \Delta\chi_i^+, {\chi^0}_i) - g(\chi^0) \tag{3}$$

where $(\chi_i + \Delta\chi_i^+, {\chi^0}_i)$ is the point obtained by shifting χ_i alone to the sensitivity case. $\Delta\psi_i^+$ are finite changes in ψ provoked by individual changes in the inputs. This method is characterized by its local one-at-a-time approach, which aligns well with the primary goal of this work, given that the experimenter determines the input variable values without incorporating uncertainty levels in the input parameters. Specifically, out of the model parameters, the dimensions and material of the cup are the ones chosen, as they can be adjusted. The sensitivity measures quantify the effect of the shift in χ_i from the base case to the sensitivity case. The insights gleaned from these sensitivity measures encompass both the magnitude of the impact and the direction of change [2].

This work is the first step in incorporating an optimization strategy to identify the optimal values of the parameters for the cup. By integrating optimization techniques into the sensitivity analysis framework, the aim is to understand the model's behavior under varying parameter values and determine the ideal parameter settings that yield the most desirable outcome. In this case, it is the attenuation of the heat flow into the beverage.

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