Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 12, n. 1, 2026.

Trabalho apresentado no XLIV CNMAC, Fundação Getulio Vargas - Rio de Janeiro - RJ, 2025.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Thermal Efficiencies of Ideal Regenerative Rankine Cycles for Water Heaters: An Approach Using Interpolation

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Abstract. This article aims to analyze and compare the thermal efficiencies of ideal regenerative Rankine cycles that use feedwater heaters exclusively, using predetermined parameters. Based on the mathematical modeling described by the equations, an attempt was made to model the systems with idealized cycle parameters and equipment, such as existing boilers and turbines. Thermodynamic tables were then used to acquire maximum and minimum temperature, entropy, and enthalpy data for the cycles under the specified conditions. Finally, the Octave program was used to calculate the thermal efficiencies of the cycles in various ranges of air removed from the turbine, and it was found that the cycle that used a closed feedwater heater (AAF) had a higher efficiency than the cycle with an open feedwater heater (AAA). The results obtained were satisfactory.

Keywords. Rankine cycle, feedwater heaters, thermodynamic cycle, thermal efficiency, interpolation.

1 Introduction

In [1] states that in thermodynamics, thermal efficiency dictates the performance of a given machine or thermodynamic cycle, demonstrating how viable it can be during operation. In the Rankine cycle, which focuses on turbine power generation, combined thermodynamic cycles are made so that the system achieves the highest possible efficiency. In the same Rankine cycle, it is possible to observe the implementation of open and closed feedwater heaters which, in cooperation, increase the cycle's efficiency. However, the question arises as to whether these heaters, analyzed from the perspective of having a cycle only for themselves, have any difference in thermal efficiency under the same operating conditions.

The article presents a guide to the introduction section. In the section 2, present model mathematical . In the section 3, present the methodology . In the section 4 present the results. In the section 5, with the conclusions are made.

2 Mathematical Model

When modeling thermodynamic models to solve problems in [1], [2], and [3], it is common to come across values for pressure, temperature, entropy, enthalpy, and specific volume that are

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DOI: 10.5540/03.2026.012.01.0340

not specified in thermodynamic tables, so it is necessary to use equations that allow interpolation between values. The following interpolation equations will be used in this article:

$$\frac{T - T_i}{T_f - T_i} = \frac{s - s_i}{s_f - s_i}, \quad \text{in} \quad \frac{KJ}{Kg \cdot K} \tag{1}$$

$$\frac{T - T_i}{T_f - T_i} = \frac{h - h_i}{h_f - h_i}, \quad \text{in} \quad \frac{KJ}{Kg} \tag{2}$$

Generally, the interpolation equations (1) and (2) are directly linked to the entropy and enthalpy equations by title of the mixture (x).

$$h = h_{\text{saturated liquid}} + x \cdot (h_{\text{saturated steam}} - h_{\text{saturated liquid}}), \quad \text{in} \quad \frac{KJ}{Kg} \tag{3}$$

$$s = s_{\text{saturated liquid}} + x \cdot (s_{\text{saturated steam}} - s_{\text{saturated liquid}}), \quad \text{in} \quad \frac{KJ}{Kg \cdot K} \tag{4}$$

Equations (3) and (4) are very important, because, in isoentropic processes, several fluids undergo a phase change (from superheated vapor to mixture) of temperature and pressure. After obtaining all the values obtained using equations (1), (2), (3), and (4), we proceed to calculate the thermal efficiency of a regenerative Rankine cycle with heaters, using the equations in the literature [2] and [3].

$$Q_{\text{input}} = \Delta h_{\text{boiler}}, \quad \text{in} \; \frac{KJ}{Kg} \tag{5}$$

$$Q_{\text{output}} = (1 - y) \cdot \Delta h_{\text{condenser}}, \quad \text{in} \quad \frac{KJ}{Kg} \tag{6}$$

$$T = y \cdot T_{\text{minimum}} + (1 - y) \cdot T_{\text{maximum}}, \quad \text{in} \quad C \tag{7}$$

$$\eta = 1 - \frac{Q_{\text{output}}}{Q_{\text{input}}}, \quad \textit{dimensionless} \tag{8}$$

With the equations mentioned above, it is possible to calculate the efficiency of most thermodynamic cycles.

3 Methodology

3.1 Optimal Operation of Regenerative Cycles With Heaters

In [2] and [3], an ideal regenerative Rankine cycle must have three pressure lines, regardless of the type of heater used: a maximum defined by the boiler which delimits the highest temperature of the cycle, a minimum defined by the condenser which determines the minimum temperature of the cycle and an intermediate defined by the turbine which determines the amount of steam extracted (y).

In the boiler, the working fluid should ideally enter as a compressed liquid and leave as superheated steam through an isobaric process, after which it goes to the turbine where part of the

DOI: 10.5540/03.2026.012.01.0340

steam is removed and guided to the open or closed feedwater heater (AAA or AAF respectively), ideally as superheated steam at an intermediate pressure, and part goes to a condenser, always through an isoentropic process [2].

In the condenser, the fluid should ideally arrive as a mixture with a minimum titer of 0.75 or as saturated vapor, so that it passes through the condenser in an isothermal process and becomes a saturated liquid. After the condenser, the fluid passes through a pump that increases its pressure to the maximum of the system and transports it isentropically to the AAA or AAF to exchange heat with the fluid separated from the turbine.

If we look at the system working with an AAA, the fluids enter the heater at different temperatures and pressures and mix ideally up to the point of saturated liquid and are then pumped isentropically, by a second pump, to the boiler inlet where the process begins again. If the system works with an AAF, the fluids enter the heater and exchange heat without mixing. The fluid leaving the condenser line is guided to a mixing chamber and the intermediate pressure fluid, after the AAF, passes through a pump that leaves it at the same pressure as the other fluid line and guides it to the mixing chamber, where both reach thermal equilibrium and enter the boiler, restarting the cycle [3].

It is worth noting that ideally in these cycles the AAA and AAF heaters and the mixing chamber do no work and do not exchange heat with the environment. Furthermore, in cycles with AAF, the portion of value removed from the turbine (y) will be responsible for directly dictating the boiler inlet temperature after the fluid has reached thermal equilibrium. Figure 1 shows the cycle with a single open feedwater heater.

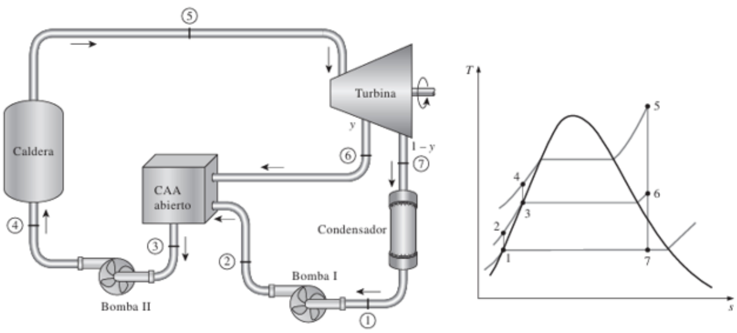

Figure 1: Cycle with a single open feedwater heater and its T-s diagram. Source: Çengel, 2012.

3.2 Definition and Calculation of Common Operating Parameters

For both cycle analysis cases, the same pumps, turbines, boilers, and the same maximum temperature and operating pressure parameters will be used. Through the analyses used in [5] and [6], the following data was chosen: Class B boiler (NR-13) with an operating pressure of 15 Bars; steam turbine for generating electricity with an inlet pressure of 15 Bars and an outlet pressure of 1.5 Bars.

It should be noted that the intermediate pressure of the extracted gas line will be 10 Bars and this extracted gas (y) will vary between 0.3 and 0.45, and the working fluid for both systems will be water. In addition, in order for the systems to be ideal, it was decided to use a fluid entering the condenser in a mixed state with a minimum titer of 0.89, to the detriment of the fluid leaving the turbine by extraction of remaining saturated liquid. According to this data, by analyzing the thermodynamic tables of [1] and [4] and using equations (1), (2), and (5), we were able to obtain the following data shown in table 1, which shows the temperature, entropy, and enthalpy values for parts of the systems.

Table 1 shows the temperature, entropy and enthalpy values for the other parts of the systems.

Table 1: Temperature, entropy and enthalpy values for parts of the systems.

	Boiler output		Condenser output		First pump output	
	min. val.	max. val.	min. val.	max. val.	min. val.	max. val.
AAA						
temperature (°C)	227.935	368.795	–	111.4	–	111.4207
entropy (kJ/kg·K)	6.6	7.1654	–	1.4336	–	1.4336
enthalpy (kJ/kg)	2868.383	3188.241	–	4.6711	–	504.812
AAF						
temperature (°C)	227.935	368.795	–	111.4	–	112.5
entropy (kJ/kg·K)	6.6	7.1654	–	1.4336	–	1.4336
enthalpy (kJ/kg)	2868.383	3188.241	–	4.6711	–	508.839

Table 2: Temperature, entropy and enthalpy values for the other parts of the systems.

	Extracted steam line		Heater output		Boiler entry	
	min. val.	max. val.	min. val.	max. val.	min. val.	max. val.
AAA						
temperature (°C)	180.625	311.772	–	179.9	–	181.899
entropy (kJ/kg·K)	6.6	7.1654	–	2.1387	–	2.1387
enthalpy (kJ/kg)	2779.872	3076.272	–	762.81	–	778.61
AAF						
temperature (°C)	180.625	311.772	–	179.9	–	181.3
entropy (kJ/kg·K)	6.6	7.1654	–	2.1119	2.13013	2.13299
enthalpy (kJ/kg)	2779.872	3076.272	–	762.81	769.887	775.99

As can be seen, most of the values remain the same throughout the process, only changing at the heaters, mixing chamber, and boiler inlet. It is worth noting that the outlet of the AAF heater has two entropy and enthalpy values, which is explained by the fact that the fluids in the equipment are not joined. These values correspond, respectively, to the entropies and enthalpies of the 10-bars and 15-bars lines.

4 Results

After acquiring the data and the conventions used for the calculations, two codes were created using Octave software to calculate the yield. The minimum and maximum enthalpy values for the two systems were used in the calculations, with the variable parameter being the amount of steam extracted from the turbine (y) from 0.3 to 0.45 with a step of 0.001 since equations (4) and (6) necessarily depend on this value.

Figure 2 shows the efficiency for a cycle with an AAA heater with minimum enthalpy.

DOI: 10.5540/03.2026.012.01.0340

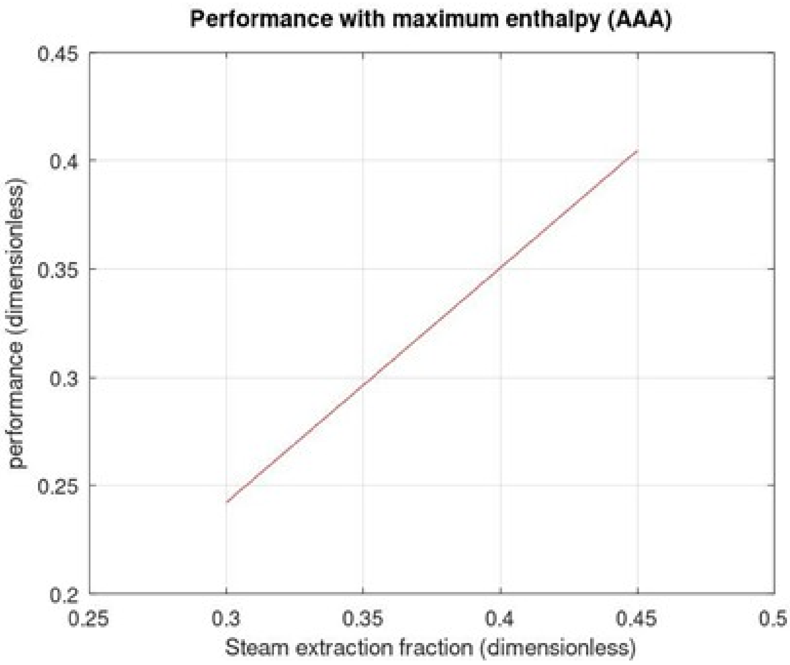

Figure 2: Performance for a cycle with an AAA heater with minimum enthalpy. Source: authors.

Figure 3 shows the efficiency for a cycle with an AAA heater at maximum enthalpy,

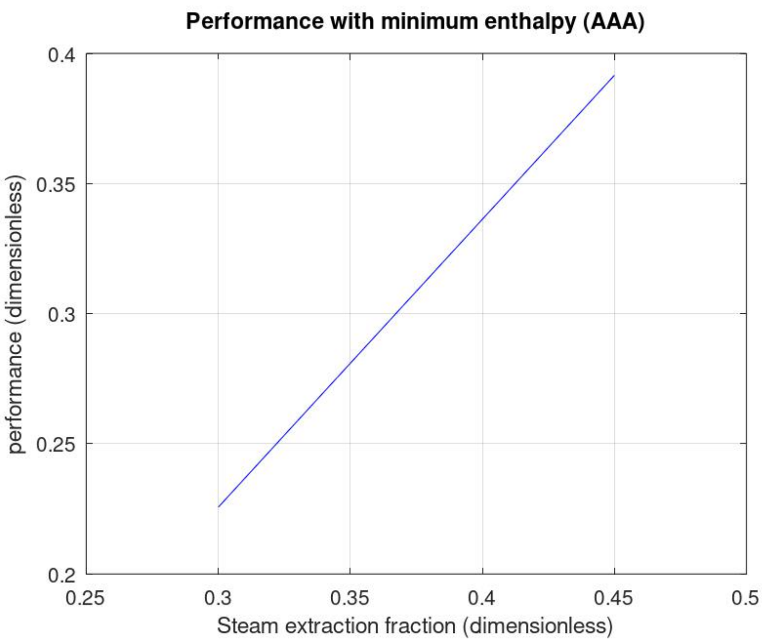

Figure 3: Performance for one cycle with AAA heater at maximum enthalpy. Source: authors.

Figure 4 shows the efficiency for a cycle with an AAF heater with minimum enthalpy,

DOI: 10.5540/03.2026.012.01.0340

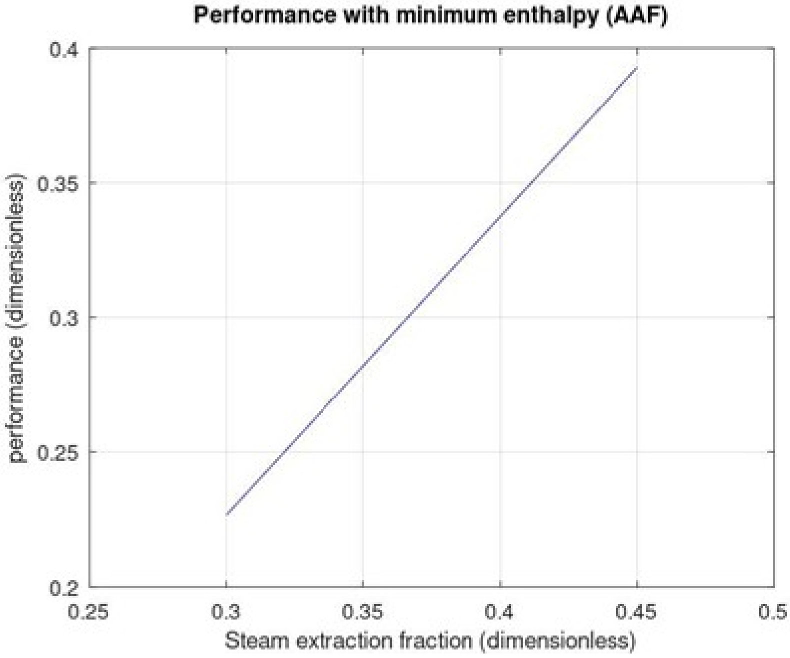

Figure 4: Performance for an AAF heater cycle with minimum enthalpy. Source: authors.

Figure 5 shows the efficiency for a cycle with an AAF heater at maximum enthalpy.

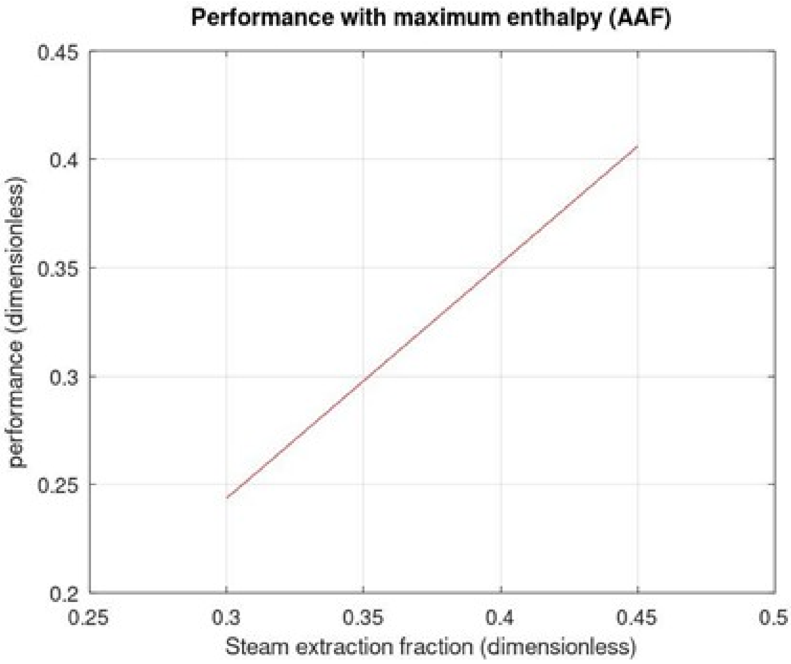

Figure 5: Performance for an AAF heater cycle with maximum enthalpy. Source: authors.

The efficiency values for the cycles operating with an AAA heater ranged from 0.2265 to 0.3912 for the minimum enthalpy values and 0.2422 to 0.4046 for the maximum enthalpy values. The cycle with an AAF heater ranged from 0.2268 to 0.3986 for the minimum enthalpy and 0.2436 to 0.406 for the maximum enthalpy. It can be seen that the values for both cases are very close, reporting an almost insignificant difference in yield for the systems built here, in the parameters specified above, along with the observation that as the value of y increases, the yield increases in both cycles.

5 Conclusions

From the analysis of the thermal efficiency values obtained, it can be said that, for the parameters chosen, the regenerative Rankine thermodynamic cycles with heaters had very close thermal yields, with the cycle with an AAF heater having a higher yield than the AAA. However, these differences are significantly irrelevant.

Likewise, it can be seen that despite the similarities in efficiency, the cycle operating with an AAF heater has a greater degree of variation in the boiler's input parameters, which are directly linked to the amount of steam extracted from the turbine, making this thermodynamic system likely to perform better than the AAA if the cycle's maximum temperature parameters are increased, the condenser's operating pressure is lowered and the amount of steam removed from the turbine is greater within an acceptable operating range and under lower pressure.

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