Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 12, n. 1, 2026.

Trabalho apresentado no XLIV CNMAC, Fundação Getulio Vargas - Rio de Janeiro - RJ, 2025.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Reliability of a Flexible Subsea Riser under Repetitive Random Loads and a Power Law Degraded Capacity

Eduardo P. Frutuoso e Melo[1]
PENO, COPPE, UFRJ, RJ
Danielle G. Teixeira[2], Paulo F. Frutuoso e Melo†[3,a], Andressa S. Nicolau[4]
PEN, COPPE, UFRJ, RJ
Maxmiano C. Martins[5]
UERJ-ZO/SENAI-CETIQT, RJ

[a]In memoriam of my beloved father, hero and mentour, Professor Paulo Fernando Ferreira Frutuoso e Melo, whose knowledge, love, patience and kindness guided me through my hardest challenges and brought me a clear guide to the future. Your legacy on the scientific community is legendary. Thank you for everything.
★ 07/07/1955 †10/08/2025

Abstract. We discuss a model that considers random loads (ship displacements, subsea currents, riser weight and fluid weight) occurring on subsea flexible risers and a degrading capacity with an initial random capacity. The number of occurrences of repetitive loads follows Poisson distributions. All loads and riser initial capacity are modeled by uniform distributions to allow analytical solutions. The degradation model is a power law.The riser reliability is the product of all contributions. Scarce field and literature data has been gathered for the initial evaluations. The riser time-dependent reliability is useful as a means for setting inspection policies.
Keywords. Subsea Risers, Random Repetitive Loads, Random Initial Capacity, Degraded Capacity, Reliability.

1 Introduction

We discuss a reliability model that considers random loads occurring on flexible risers in subsea applications. It is assumed that the numbers of occurrence of these loads follow Poisson distributions. There are two repetitive loads: a) Floating Production, Storage and Offloading (FPSO) unit displacements caused by waves, surface currents and wind; b) induced by subsea currents.There are two further loads; c) flexible riser weight; d) weight of the fluid transferred from the sea bottom to the FPSO unit and vice-versa (e.g.,crude oil or water injection). It is assumed that all these loads are independent and modeled by known probability distributions. To allow for some preliminary analytical results, all loads and initial capacity follow uniform probability distributions. It is also assumed that the initial riser capacity degrades over time due to operational conditions. The degradation model is a power law, which considers three parameters, [1]: a shape parameter, a scale parameter and a parameter related to initial capacity loss. The riser reliability is the product of all four contributions. Some field and literature data has been gathered for the evaluations. The riser reliability as a time function is useful for considerations of equipment degradation and also

[1]eduardo.melo@oceanica.ufrj.br
[2]dteixeira@nuclear.ufrj.br
[3]frutoso@nuclear.ufrj.br
[4]andressa@lmp.ufrj.br
[5]maximiano.martins@uerj.br

for discussing possible inspection and maintenance policies. It is assumed that the riser design life is 20 yr [2].
Although the proposed model does not consider dependencies between loads and resistance [3], it presents features that have not been approached so far for the problem at hand.
This paper is organized as follows. Section 2 addresses the loads the riser is subject to. The capacity model is discussed in Section 3. Section 4 presents results obtained and Section 5 displays the conclusions reached and recommendations for future work.

2 Methodology

2.1 General Considerations

To obtain the reliability R of a flexible riser we consider that it is subject to a random load L that is applied once and its capacity C is also a random variable. If L and C are independent, one can write Eq.(1) (f_L and f_C are the probability densities of load and capacity, respectively), [1]:

$$R = \int_0^{\infty} f_L(l) \left[\int_l^{\infty} f_C(c) dc \right] dl \tag{1}$$

We have to model two kinds of loads: those that are repetitive (e.g., wind loads) and those that are permanent (as dead load, or weight load), so time must be explicitly considered. Moreover, the riser capacity degrades with time and its initial value follows a probability distribution.

2.2 Modeling Degrading Capacity

The riser capacity is modeled over time to consider failure mechanisms related to the environment where risers are installed (deep water). Oil fields are rich in corrosive gases, which together with top high tensions and salt water can create an accelerated corrosion on the steel wires that sustain the riser structure, [7]. In this sense, we consider a power law [1]:

$$c(t) = c_o \left[1 - \alpha \left(t/t_o\right)^m\right] \tag{2}$$

where $c(t)$ is the riser capacity, c_o is its initial value ($t = 0$), m is a shape parameter, to is a scale parameter and α is a parameter related to the loss of initial capacity. c_o is a random variable that follows a probability distribution.
The fit of the capacity data measured in inspections to the model presented in Eq.(2) can be developed by transforming Eq.(2) to a straight line [1], Eq.(3):

$$\ln r(t) = m. \ln t + \ln \alpha - m. \ln t_o \tag{3}$$

where:

$$r(t) = 1 - c(t)/c_o \tag{4}$$

Considering a straight-line $y = ax + b$, its slope will be given by $a = m$ and its intercept by $b = \ln(\alpha) - a \ln(t_o)$: The line fitting gives us two coefficients, but we have three parameters to estimate. Therefore, we need a third equation. Note that from Eq.(3):

$$\ln r(t_o) = \ln \alpha \tag{5}$$

The area of trapezoid ABCD is equal to the sum of the areas of trapezoids AEFD and EBCF, Fig.1. The parameters to uniquely characterize Eq.(3) are given by Eqs.(6) and (7):

$$t_o = \exp \left(\frac{\ln r(t_m). \ln t_M - \ln r(t_M). \ln t_m - b(\ln t_M. \ln t_m)}{\ln r(t_m) - \ln(t_M) + a(\ln t_M. \ln t_m)} \right) \tag{6}$$

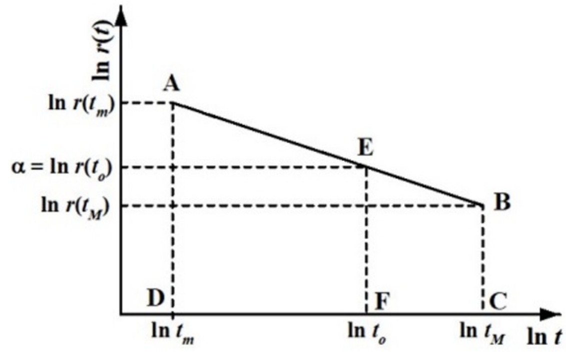

Figure 1: Estimation of parameter t_0. Source: Author's own.

$$\alpha = \exp\left(b + \frac{\ln r(t_m).\ln t_M - \ln r(t_M).\ln t_m - b(\ln t_M.\ln t_m)}{\ln r(t_m) - \ln(t_M) + a(\ln t_M.\ln t_m)}\right) \tag{7}$$

2.3 Modeling Constant Loads

The constant loads to be modeled are related to the riser weight and also to the fluid transported by it. The load due to the riser weight is modeled by a uniform distribution, Eq.(8):

$$f_{L_W}(l_W) = (l_{W,M} - l_{W,m})^{-1}, l_{W,m} \leq l_W \leq l_{W,M} \tag{8}$$

The riser initial capacity, C_0, is also modeled by a uniform distribution, Eq.(9):

$$f_{C_o}(c_o) = (c_{o,M} - c_{o,m})^{-1}, c_{o,m} \leq c_o \leq c_{o,M} \tag{9}$$

Given an initial capacity c_0, the riser reliability considering its weight can be found from Eq.(10):

$$R_W(t|c_o) = \int_{l_{W,m}}^{c(t)} f_{l_W}(l_W) dl_W \tag{10}$$

where, $R_W(t|c_o)$ is the conditional reliability related to the riser weight given an initial capacity equal to c_o. Putting Eqs.(2) and (8) in Eq.(10) one obtains:

$$R_W(t|c_o) = \frac{c_o\left[1 - (t/t_o)^m\right] - l_{W,m}}{l_{W,M} - l_{W,m}} \tag{11}$$

Finally, the reliability related to the riser weight is given by Eq. (12):

$$R_W(t) = \int_{c_{o,m}}^{c_{o,M}} f_{C_o}(c_o) R_W(t|c_o) dc_o \tag{12}$$

By putting Eq.(9) and Eq.(11) in Eq.(12) one has:

$$R_W(t|c_o) = \frac{\mu_{C_o}\left[1 - (t/t_o)^m\right] - l_{W,m}}{l_{W,M} - l_{W,m}} \tag{13}$$

where the mean of the initial riser capacity is given by:

$$\mu_{C_o} = 0.5(c_{o,m} + c_{o,M}) \tag{14}$$

DOI: 10.5540/03.2026.012.01.0333

As $f_{L_W}(l_W)$ is a probability density

$$\frac{\mu_{C_o}}{l_{W,m}} \leq \frac{1}{1-(t/t_o)^m} \tag{15}$$

The reliability related to the fluid transported by the riser, RT (t), is given by Eq. (16):

$$R_T(t) = \frac{\mu_{C_o}\left[1-(t/t_o)^m\right] - l_{T,m}}{l_{T,M} - l_{T,m}} \tag{16}$$

A mean defined as in Eq. (14) is valid here, as long as the inequality in Eq. (15) for $l_{T,M}$ holds.

2.4 Modeling Repetitive Loads [1]

The probability p of a single load causing an equipment to fail is given by Eq. (17):

$$p = \int_{c(t)}^{\infty} f_L(l)dl \tag{17}$$

For repetitive loads, let us assume that they occur at a constant rate γ and the number of occurrences follows a Poisson distribution [1]. Therefore, γp is the fraction of loads that cause the riser failure. As the equipment fails only once, then:

$$R(t+\Delta t) = \left[1-\gamma p \Delta t\right] R(t) \tag{18}$$

However, $c(t)$ depends on the initial capacity for which we are assuming a variability, so:

$$R(t|c_o) = \exp\left[-\int_0^t dt'.\gamma \int_{c(t')}^{\infty} f_L(l)dl\right] \tag{19}$$

Using again Eq. (12) (without index W), we finally have:

$$R(t) = \int_0^{\infty} f_C(c_o) \exp\left[-\int_0^t dt'.\gamma \int_{c(t')}^{\infty} f_L(l)dl\right] dc_o \tag{20}$$

2.5 Composite Model for Loadings and Time-Dependent Capacity

The model for evaluating the time-dependent reliability of the flexible riser is given by:

$$R(t) = R_W(t)R_T(t)R_F(t)R_U(t) \tag{21}$$

where $R_W(t)$ is the reliability related to its weight, $R_T(t)$ is the reliability related to the fluid it transports, $R_F(t)$ is the reliability related to FPSO displacements and $R_U(t)$ is the reliability related to subsea currents. All loads and the riser capacity are assumed independent. $R_W(t)$ is given by Eq. (11) and $R_T(t)$ by Eq. (22):

$$R_T(t) = \frac{\mu_{C_o}\left[1-(t/t_o)^m\right] - l_{T,m}}{l_{T,M} - l_{T,m}} \tag{22}$$

Notice that a probability density for lT is defined much in the same way as in Eq.(8). For the case of $R_F(t)$, one has (from Eq.(20)):

$$R_F(t) = \frac{(l_{F,M} - l_{F,m})K_F^{-1}(t)}{l_{F,M} - c_{o,m}} e^{-\frac{\gamma_F t l_{F,M}}{l_{F,M}-l_{F,m}}} \left[e^{\frac{l_{F,M}}{l_{F,M}-l_{F,m}}K_F(t)} - e^{\frac{c_{o,m}}{l_{F,M}-l_{F,m}}K_F(t)}\right] \tag{23}$$

where:

$$K_F(t) = \gamma_F t \left[1 - \alpha(m+1)^{-1}(t/t_o)^m\right], \tag{24}$$

and γ_F is the rate of occurrence of FPSO displacements and:

$$f_{L_F}(l_F) = (l_{F,M} - l_{F,m})^{-1}, l_{F,m} \leq l_F \leq l_{F,M} \tag{25}$$

Finally, for $R_U(t)$ one has:

$$R_U(t) = \frac{(l_{U,M} - l_{U,m})K_U^{-1}(t)}{l_{F,M} - c_{o,m}} e^{-\frac{\gamma_U t l_{U,M}}{l_{U,M} - l_{U,m}}} \left[e^{\frac{l_{U,M}}{l_{U,M} - l_{U,m}} K_U(t)} - e^{\frac{c_{o,m}}{l_{U,M} - l_{U,m}} K_U(t)} \right] \tag{26}$$

where $K_U(t)$ is defined as $K_F(t)$ (Eq. (24)), γ_F is replaced by γ_U and $F_{L_U}(l_U)$ is defined as in Eq.(25).

3 Data and Local Sensitivity Analysis on Riser Capacity

Fig.2(a) displays the capacity as a function of time for the 20-year riser design life as a function of parameter α. Clearly, $\alpha \geq 0.1$, which means a maximum loss of around 50% of the initial capacity. Fig.2(b) illustrates the riser capacity as a function of m, which should be small for the same reason as for α. Fig. 2(c) displays the riser capacity behavior as a function of t_o. For $t_o < 5$ yr, the riser capacity falls quite quickly, so it is advisable that $t_o = 5$ yr at least. Finally, Fig. 2(d) displays the riser capacity as a function of c_o ($t_F = \text{ton} - \text{force}$).

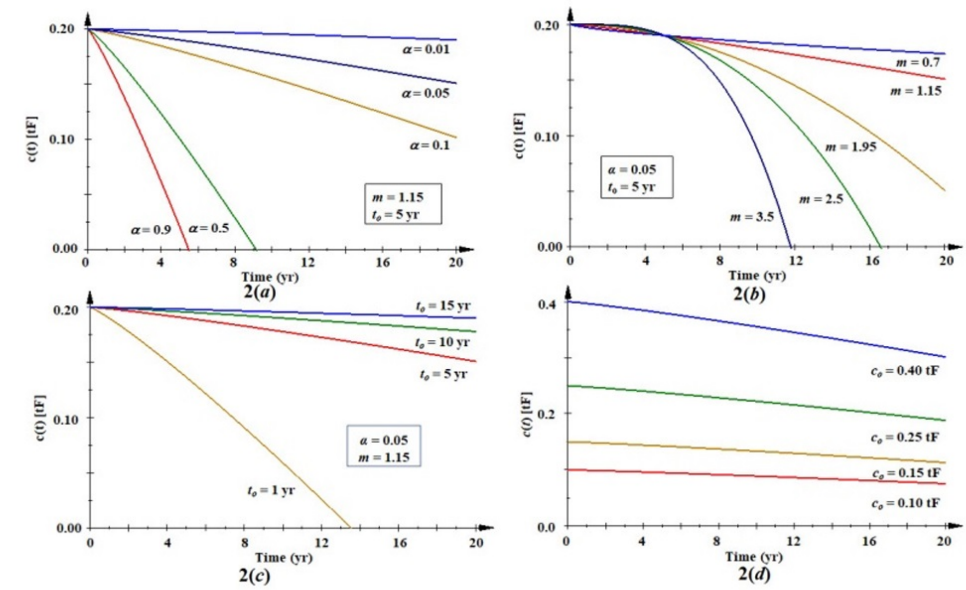

Figure 2: Riser capacity as a function of α (a), m (b), t_o (c), and c_o (d). Source: Author's own.

Based on the above, we adopted the values shown in Table (1) due to the lack of field data. The load data used is presented in terms of means and standard deviations (SD). The coefficients of variation are 0.10 [6]. As to the load due to subsea currents we considered that the mean of these loads is slightly less than that due to FPSO displacements. We also considered that the riser initial resistance mean is equal to that of FPSO displacements, based on our field experience. Likewise, the riser weight load mean was set equal to that of the load related to the liquefied petroleum gas (LPG) transfer. In order to define the uniform distributions, Table (1) displays the minimum and maximum value for each case. Table (1) also displays the assumed rates of occurrence of loads (Poisson).

Table 1: Input data for the analysis

Parameter	Value	Parameter	Value
α	0.01	$\gamma_F(yr^{-1})$	0.4
m	1.15	$\gamma_U(yr^{-1})$	0.2
$t_o(yr)$	7	$\gamma_U(yr^{-1})$	0.2

Variable	Mean	SD	Min(t_F)	Max(t_F)
Riser initial resistance (t_F)	317	31.7	262.1	371.9
FPSO displacements (t_F)	317	31.7	262.1	371.9
Underwater sea currents (t_F)	250	25.0	206.7	293.3
Riser weight load (t_F)	224	22.4	185.2	262.8
Fluid transfer (LPG) (t_F)	224	22.4	185.2	262.8

4 Results and Discussion

Fig. 3(a) displays all reliability curves. The curves for $R_W(t)$ and $R_U(t)$ superimpose because their parameters are quite the same. It also displays the riser reliability (Eq.(21)). Fig. 3(a) shows that the greatest reliability loss within the riser design life is the one related to FPSO displacements and this is to be expected. As displayed in Fig. 3(a), the riser reliability decay is significant mainly because of the contribution from FPSO displacements. The riser capacity degradation has a significant role also. An important possibility is the consideration of periodic inspection. This issue has concerned the field for a long time [5].
Fig. 3(b) shows alternatives for improving riser reliability. The riser mean time to failure (MTTF) is the area under the reliability curve [1]; a comparison of the areas in Figs. 3(a) and 3(b) reveals that this information can be useful for inspection optimization purposes. Fig. 3(b) shows that the initial riser reliability after inspection is normally a little lower than the one of the last inspection (or the value at $t = 0$, as shown) because inspection / maintenance might be imperfect.

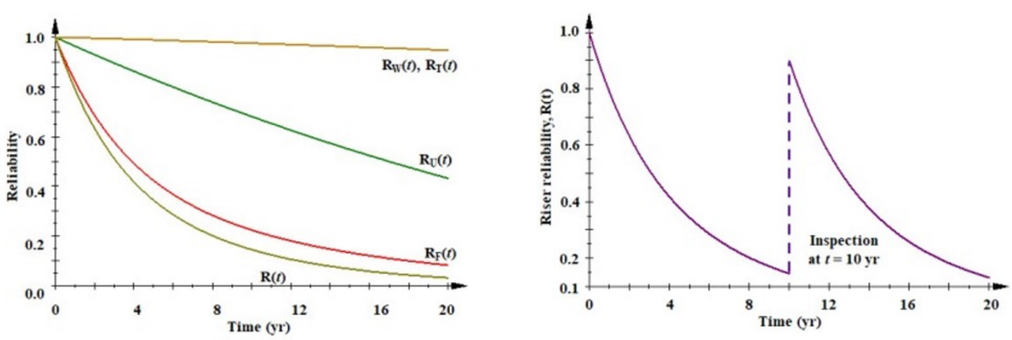

Figure 3: Contributions for the riser reliability and overall riser reliability (a) and reliability for riser inspection at half life (b). Source: Author's own.

5 Conclusions

The preliminary model assumed for the initial capacity and for all loads is a uniform distribution because it is possible to obtain analytical solutions and get insights into the problem. It is more realistic to model them by means of normal distributions [3], in which case, we would not obtain analytical solutions.
Another assumption adopted is that all variables were considered independent, which is not a reasonable assumption in practice, [3] for example, the greater the FPSO displacement, the greater the dynamic load on the riser, the greater the load, the greater the chances of failure at a fragile point, etc. In this sense, it is advisable to use multivariate analysis methods [3].
It is necessary to gather field data on riser capacity.
For other probability distributions assumed, it would be necessary to obtain Eq. (21) by means of numerical methods, e.g., compound Simpson rule, [4].
Also, a global sensitivity analysis on the parameters should be developed.

Acknowledgement

DGT, PFFM, and ASN acknowledge support from FAPERJ through grants E-26/203.489/2023, E-26/204.363/2024, and E-26/204.565/2024, respectively. Additionally, PFFM acknowledges support from CNPq through grants 406822/2022-0 and 311911/2023-4.

References

[1] E. E. Lewis. **Introduction to Reliability Engineering**. 2a. ed. New York: Wiley, 1996.

[2] E. R. Malta. “Flexible Riser Design for Deep Water”. (in Portuguese). Master dissertation. São Paulo: USP, 2010.

[3] R. E. Melchers and A. T. Beck. **Structural Reliability Analysis and Prediction**. Hoboken, NJ: Wiley, 2017.

[4] S. Nakamura. **Computational Methods in Engineering and Science with Applications to Fluid Dynamics and Nuclear Systems**. 1a. ed. New York: Wiley, 1997.

[5] J. M. M. Out, D. A. Kornemeijer, P. J. van de Loo, and A. de Sterke. “The integrity of flexible pipe: search for an inspection strategy”. In: **Engineering Structures** (1995), pp. 305–314.

[6] S. S. Rao. **Reliability-Based Design**. New York: McGraw-Hill, 1992.

[7] H. Xianwei, Z. Canfeng, D. Menglan, and A. Chen. “Reliability analysis of marine risers with narrow and long corrosion defects under combined loads”. In: **Petroleum Science** 11 (2014), pp. 139–146. DOI: 10.1007/s12182-014-0325-6.

