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Combustion Model in a Multilayer Porous Medium: Quasi-Linear Problem

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Combustion processes in porous media occur in many important applications, such as in-situ oil reservoirs and biogas production in sanitary landfills. In oil recovery, the porous medium where combustion takes place consists of layers with different physical properties. Understanding how temperature changes during and after the combustion process is important for making decisions about the oil recovery process. We analyze a one-dimensional reaction-diffusion-convection model, in which temperature and gas concentration are the unknowns in the layers.

A model for two layers was derived by Da Mota and Schecter [6], who identified and analyzed a family of traveling wave solutions. Da Mota and Santos [7] later proved the existence of a unique global solution to the associated Cauchy problem using the monotone iterative method, under specific fuel concentration conditions. This study was extended to a broader fuel concentration range in Da Mota et al. [5] and to multilayer systems in Batista et al. [3, 4]. A different approach was taken by Alarcon et al. [1] and Batista et al. [2], using abstract semigroup theory in Hilbert spaces.

The present study examines the model derived in [3].

$$\begin{cases} (u_i)_t - \alpha_i(y_i)(u_i)_{xx} + \beta_i(y_i)(u_i)_x = f_i(u, y_i), & x \in \mathbb{R}, \quad t > 0, \\ (y_i)_t = -A_i y_i g(u_i), & x \in \mathbb{R}, \quad t > 0, \\ \big(u_i(x,0), y_i(x,0)\big) = \big(\phi_i(x), y_{i,0}(x)\big), & x \in \mathbb{R}\,. \end{cases} \tag{1}$$

Here, $u_i = u_i(x,t)$ and $y_i = y_i(x,t)$ represent the temperature and fuel concentration in layer i, respectively, where $i = 1, \cdots, n$. The initial conditions for temperature and fuel concentration are denoted by ϕ_i and $y_{i,0}$, respectively. In vector form, we write $u = (u_1, \cdots, u_n)$, $y = (y_1, \cdots, y_n)$.

The source functions are:

$$\begin{aligned} f_1(u, y_1) &= \frac{(b_1 A_1 u_1 + d_1) y_1 g(u_1)}{a_1 + b_1 y_1} + \frac{q_1}{a_1 + b_1 y_1}(u_2 - u_1) - \frac{\overline{q}_1}{a_1 + b_1 y_1}(u_1 - u_e), \\ f_i(u, y_i) &= \frac{(b_i A_i u_i + d_i) y_i g(u_i)}{a_i + b_i y_i} - \frac{q_{i-1}}{a_i + b_i y_i}(u_i - u_{i-1}) + \frac{q_i}{a_i + b_i y_i}(u_{i+1} - u_i), \\ f_n(u, y_n) &= \frac{(b_n A_n u_n + d_n) y_n g(u_n)}{a_n + b_n y_n} - \frac{q_{n-1}}{a_n + b_n y_n}(u_n - u_{n-1}) - \frac{\overline{q}_2}{a_n + b_n y_n}(u_n - u_e), \end{aligned} \tag{2}$$

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Here, $2 \leq i \leq n-1$, a_i, b_i, c_i, d_i, A_i, λ_i, q_i, $\bar{q}_1$, $\bar{q}_2$, and E, are all non-negative functions of the spatial variable x, depending on the physical properties of the gas, fuel, and rock. The external environment temperature is denoted by u_e. Notably, a_i and λ_i are strictly positive. The function g, based on the Arrhenius reaction law, is defined as: $g(\theta) = \begin{cases} e^{-\frac{E}{\theta}}, & \text{if } \theta > 0 \\ 0, & \text{if } \theta \leq 0\,. \end{cases}$

The coefficient functions α_i and β_i are defined by $\alpha_i(y_i) = \frac{\lambda_i}{a_i + b_i y_i}$, and $\beta_i(y_i) = \frac{c_i}{a_i + b_i y_i}$.

Specifically, we prove that the problem has a local solution, as stated in the following theorem:

Theorem 0.1 (Local solution)**.** *Let $s \geq 2$. Then the initial value problem* (1) *has a unique solution denoted by $u = (u_1, \ldots, u_n) \in C\big([0, T], \mathtt{H}^s(\mathbb{R})^n\big)$, for some $T > 0$, provided that $\phi = (\phi_1, \ldots, \phi_n) \in \mathtt{H}^s(\mathbb{R})^n$. Here $y_i = y_i(u_i)$.*

We continue working to prove the existence of global solutions, their continuous dependence on the data, and to obtain numerical simulations (see Figure 1).

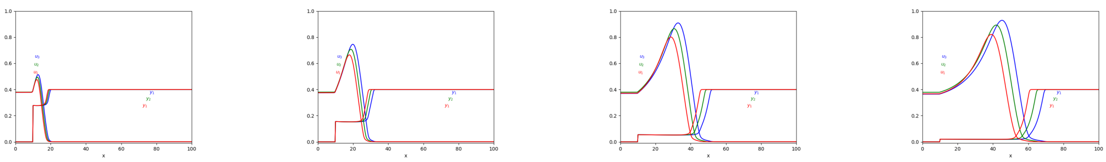

Figure 1: Evolution of temperatures and concentrations over time. Source: authors.

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