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Multi-objective Optimization with Genetic Algorithms for the Release of Wolbachia Mosquitoes Models

César Vian[1] Pastor Perez[2] Diego P. Pinto-Roa[3] Francisco Benitez[4] Christian Schaerer[5]
Polytechnic Faculty, UNA, Paraguay

Wolbachia is a bacterium that reduces the ability of mosquitoes to transmit arboviruses. Vector control by releasing Wolbachia-infected mosquitoes is a promising strategy to reduce the transmission of diseases such as dengue. This work uses a two-sex mathematical model (see [2]) to analyze releases combining suppression (releasing infected males) and replacement (releasing infected females) strategies for Wolbachia-infected mosquitoes. Thus, we formulated the following problem: At what time intervals and density should male and female mosquitoes infected with Wolbachia be released to ensure optimal intervention? To answer this question, we applied a multi-objective optimization algorithm based on genetic algorithms to achieve replacement with the least number of mosquitoes in the shortest possible time.

The mosquito population dynamics are governed by releases of infected males and females, where $v_{L\gamma}$ and $v_{A\gamma}$ represent the input variables. This yields the following controlled system:

$$\dot{A} = m(|A|, s)\,\alpha(A) - \mu(|A|, s)A + V, \quad A^{\mathrm{T}} = (A_M^U \;\; A_F^U \;\; A_M^W \;\; A_F^W) \tag{1}$$

$$m(|A|) := \mathrm{diag}\{m_\gamma^\eta(|A|)\},\ \mu(|A|) := \mathrm{diag}\{\mu_\gamma^\eta(|A|)\}, \quad V = (0, 0, v_M, v_F), \tag{2}$$

$$\alpha(A_M, A_F) := \sum_{\eta=U,W} \sum_{\gamma=M,F} \alpha_\gamma^\eta(A)(e^\eta \otimes e_\gamma),\ \alpha_\gamma^\eta(A_M, A_F) := \frac{1}{|A|} A_{\gamma'}^T G^\eta A_\gamma, \quad \gamma' \neq \gamma \tag{3}$$

$$G^U = \begin{pmatrix} 1 & 0 \\ 1-\sigma & 0 \end{pmatrix}, \quad G^W = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad \delta_\eta^{\eta'} := \begin{cases} 1, & \text{if } \eta = \eta', \\ 0, & \text{if } \eta \neq \eta' \end{cases} \tag{4}$$

Population dynamics parameters: Recruitment rate ($r^\eta \alpha_\gamma^\eta(A)$) determines new larval production based on adult density. Mortality rates ($\mu_\gamma^\eta(L)$, $\mu_\gamma^\eta(A)$) for larvae (L) and adults (A). Cytoplasmic incompatibility (σ): when infected males (W) with uninfected females (U), $\sigma = 1$ causes complete sterility. Invasion indicators (δ_η^m): Kronecker delta ($\delta_\eta^m = 1$ if $\eta = W$, else 0).

State variables: A: adult mosquito density vector; A_M^U: uninfected adult males; A_F^U: uninfected adult females; A_M^W: Wolbachia-infected adult males; A_F^W: Wolbachia-infected adult females.

Dynamics: $\dot{A}$: rate of change of adult density; $m(|A|, s)$: recruitment rate; $\alpha(A)$: trait inheritance function; $\mu(|A|, s)$: mortality rate; V: release of Wolbachia-infected mosquitoes (control).

Seasonality: $s(t) = 22.60 + 4.67 \cdot \cos\left(\frac{2\pi t}{365}\right) + 1.08 \cdot \sin\left(\frac{2\pi t}{365}\right)$ (parameters detailed in `https://doi.org/10.5281/zenodo.15277660`).

The multiobjective function of quality Q and cost C (Figure 1) is defined as:

$$\min\left\{ \; Q(A) := \tfrac{A_F^U + 1}{A_F^W + 1}, \quad C(p_M, p_F, T_M, T_F, T_M^0, T_F^0) := \textstyle\sum_{\gamma=M,F} (T_\gamma + 1) p_\gamma k_\gamma \; \right\} \tag{5}$$

[1] cesarvianr@gmail.com
[2] peperez.estigarribia@pol.una.py
[3] dpinto@pol.una.py
[4] benitez.fj.94@gmail.com
[5] cschaer@pol.una.py

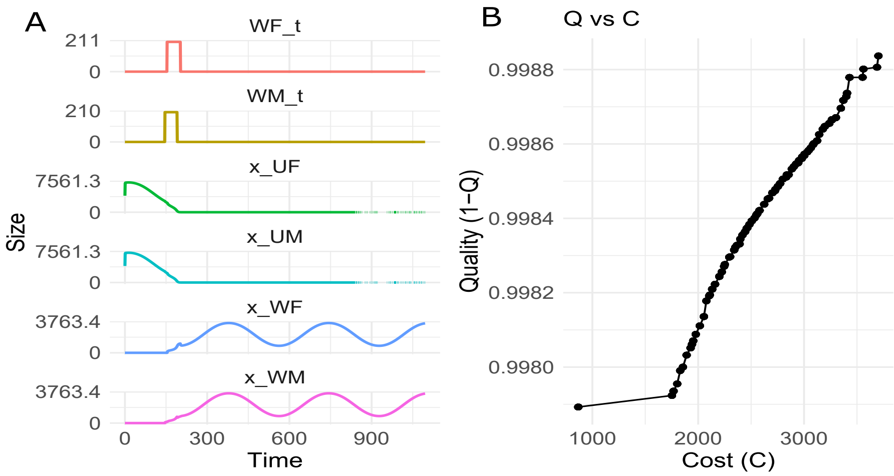

Figure 1: A. A simulation of NSGA-II solution B. Q vs C of non-dominated. Source: Authors own work

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) [1] is employed for multi-objective optimization in mosquito control, following its successful application in similar contexts [3]. This algorithm is particularly suitable due to its ability to generate well-distributed solutions along the Pareto front.

The optimization objective is to achieve Wolbachia invasion using the minimal number of mosquitoes in the shortest time frame. The controlled system is described by $\dot{A} = f(A, \Theta(t), U, \Omega)$, where: - A represents the state variables, - $\Theta(t)$ denotes time-dependent parameters, - U is the control function, - Ω comprises the optimization variables for control parameters.

The parameter space for NSGA-II optimization is defined as: $\Omega = \{p_M,\ p_F,\ T_M^0,\ T_F^0,\ T_M,\ T_F\}$, where: - p_M: number of males released over time - p_F: number of females released over time - T_M^0: initial male release time - T_M: final male release time - T_F^0: initial female release time - T_F: final female release time

Conclusion — The results (Fig. 1) demonstrate that releasing Wolbachia-infected males effectively suppresses the uninfected mosquito population density, consequently reducing the number of infected females required to achieve population replacement. The NSGA-II genetic algorithm successfully identified control parameters that validate the hybrid suppression-replacement strategy as a viable approach for Wolbachia-based vector control.

References

[1] Evolutionary Optimization Laboratory. **rmoo: Multi-Objective Optimization in R**. Online. Accessed on 10/02/2025. URL: `https://github.com/Evolutionary-Optimization-Laboratory/rmoo`.

[2] P. E. Pérez and C. E. Schaerer. "A Two-Sex Mathematical Model for Mosquito Vector Control by Wolbachia Infection". In: **MEDTROP 2024, 59° Congresso da Sociedade Brasileira de Medicina Tropical**. Sept. 2024. DOI: `10.13140/RG.2.2.22673.65122`.

[3] A. S. V. de Vasconcelos, J. S. de Lima, and R. T. N. Cardoso. "Multiobjective optimization to assess dengue control costs using a climate-dependent epidemiological model". In: **Scientific Reports** 13 (2023), pp. 10271–10289. DOI: `10.1038/s41598-023-36903-w`.