Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 12, n. 1, 2026.

Trabalho apresentado no XLIV CNMAC, Fundação Getulio Vargas - Rio de Janeiro - RJ, 2025.

Advancing Portfolio Optimization through Quantum Computing

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Quantum computing is a revolutionary technology that introduces a new computational framework and offers the potential for exponential or polynomial speedups in solving problems that are currently infeasible for classical systems. Among its most promising applications are challenges in combinatorial optimization [4, 5], particularly in finance [1, 2]. This research aims to explore the potential of quantum computing to improve portfolio optimization in finance by leveraging advanced quantum algorithms. It focuses on refining the Quadratic Unconstrained Binary Optimization (QUBO) formulation, a model used for portfolio optimization, and testing its performance using quantum variational algorithms, such as the Quantum Approximate Optimization Algorithm (QAOA) [5] and the Variational Quantum Eigensolver (VQE) [6].

The accompanying repository [3], developed as part of this research, implements QAOA to optimize the cost function defined in (1) and benchmarks its performance against a classical optimization method for maximizing the Sharpe ratio [7]. The quantum approach, implemented in 'notebooks/qaoa.ipynb', formulates the problem as a Quadratic Unconstrained Binary Optimization (QUBO) task and constructs a Hamiltonian whose ground state encodes the optimal solution for (1). Simultaneously, the classical method, implemented in 'notebooks/classical_optimization.ipynb', calculates optimal portfolio weights to maximize the Sharpe ratio. Both methods are applied to the same dataset: QAOA selects two assets for inclusion in the optimal portfolio because the budget constraint B, defined in (1), was set to be 2, while the classical method returns a weighted portfolio. Notably, the two assets selected by QAOA align with the top two assets identified by the classical approach, demonstrating the consistency of the quantum algorithm with classical optimization techniques.

The QUBO formulation for portfolio optimization involves minimizing the following cost function:

$$C(x) = -\sum_i r_i x_i + q \sum_{i>j} C_{ij} x_i x_j + \lambda(B - \sum_i x_i)^2, \tag{1}$$

where x represents the binary vector of asset inclusion, C is the covariance matrix of asset returns, r is the expected returns vector, nd q is a risk factor that governs the balance between risk and return. The budget constraint is represented by B, and λ is a regularization parameter that controls the impact of the budget constraint. This formulation minimizes the portfolio risk while optimizing returns under a budget constraint, a standard objective in financial portfolio optimization. A detailed description of this methodology can be found in [2].

The cost function (1) integrates quadratic and linear terms to represent the portfolio's risk, expected return, and constraints. However, the interplay of these terms presents challenges, particularly in interpreting the parameters q and λ, which govern risk preferences and constraint penalties, respectively.

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While the implementation showed promising results when applied to a dataset with significant variation in asset returns, it encountered limitations with datasets where returns were more closely aligned, and covariance played a more significant role in optimization. In such cases, adjusting the parameters λ and q presented several challenges.

To address these challenges, this research explores refinements to the QUBO formulation, focusing on improving the interpretability and scalability of the cost function. Adjustments aim to ensure a better balance between the quadratic and linear terms, enabling intuitive parameter definitions and resulting in a coherent and practical model to guide precise investment decisions.

Moving forward, this research will focus on exploring various QAOA versions and optimizing its initial hyperparameters, as suggested in [1]. The objective is to identify the most effective approach for optimizing the refined QUBO formulation.

Acknowledgments

This work is supported by FAPEMIG.

References

[1] S. Brandhofer, D. Braun, V. Dehn, G. Hellstern, M. Huls, Y. Ji, I. Polian, A. S. Bhatia, and T. Wellens. "Benchmarking the performance of portfolio optimization with QAOA". In: **Quantum Information Processing** 22 (2022). URL: https://api.semanticscholar.org/CorpusID:250920060.

[2] A. Canabarro, T. Mendonça, R. Nery, G. Moreno, A. Albino, G. F. de Jesus, and R. Chaves. "Quantum Finance: um tutorial de computação quântica aplicada ao mercado financeiro". In: **Revista Brasileira de Ensino de Física** 44.2 (2022).

[3] L. Costa. **Quantum Portfolio Optimization: GitHub Repository**. Online. Accessed on 24/01/2025, https://github.com/luan521/quantum-portfolio-optimization.

[4] M. Dupont, B. Evert, M. J. Hodson, B. Sundar, S. Jeffrey, Y. Yamaguchi, D. Feng, F. B. Maciejewski, S. Hadfield, M. S. Alam, Z. Wang, S. Grabbe, P. A. Lott, E. G. Rieffel, D. Venturelli, and M. J. Reagor. "Quantum-enhanced greedy combinatorial optimization solver". In: **Science Advances** 9.45 (2023).

[5] E. Farhi, J. Goldstone, and S. Gutmann. **A Quantum Approximate Optimization Algorithm**. 2014. arXiv: 1411.4028 [quant-ph]. URL: https://arxiv.org/abs/1411.4028.

[6] A. Peruzzo, J. McClean, P. Shadbolt, et al. "A variational eigenvalue solver on a photonic quantum processor". In: **Nature Communications** 5 (2014), pp. 4213.

[7] W. F. Sharpe. In: **The Journal of Business** 1 (1966). ISSN: 00219398, 15375374. URL: http://www.jstor.org/stable/2351741.

