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Langevin Equation Based on Deformed Derivatives

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In this contribution we can consider that the dynamical evolution of the granular system follows possible anomalous dynamics, characterized by different dynamical equations and with the presence of dissipation intrinsically. By this justification, we generalize the Langevin Equations (LE), to describe granular gases dynamics as dissipative systems and, for such intend we consider different forms of deformed derivatives (DD) as derivatives which are included in the kinetic equations. As a consequence of this description, the geometry of phase-space, implicit in the choice of DD by the mapping to fractal continuous, has deep influence in the form of the solutions for the corresponding deformed LE. We claim that the dynamical evolution of the granular system follows possible anomalous dynamics, characterized by different dynamical equations and with the presence of dissipation intrinsically. By this justification, we generalize the Langevin Equations (LE), to describe granular gases dynamics as dissipative systems and, for such intend we consider different forms of DD as derivatives which are included in the kinetic equations.

Here we also claim, as in Ref. [7], that new conceptions and approaches, such as DD, may allow us to understand new systems. In particular, the use of deformed derivatives (local), similarly to the (nonlocal) fractional calculus (FC), allows us to describe and emulate complex dynamics involving environmental variation, without the addition of explicit terms relating to this complexity in the dynamical equations describing the system, i.e, without explicit many-body, dissipation or geometrical terms [5].

Here by using the conformable derivative form of DD for differentiable functions, see Refs. [5–7], we can write the conformable Langevin-like equation without noise term $\eta\left(t\right)$ as

$$D_t^{\alpha} v(t) \quad = \quad -\lambda v(t), \tag{1}$$

The solution again is obtained directly from a simple integration and is given in terms of a stretched exponential as

$$v(t) \quad = \quad v_0 e^{\frac{-\lambda t^{\alpha}}{\alpha}}. \tag{2}$$

Now we follow by considering dual conformable derivative. In this case, the Langevin-like equations without noise term $\eta\left(t\right)$assumes the form

$$\tilde{D}_t^{\alpha} v(t) \quad = \quad -\lambda v(t). \tag{3}$$

Substituting the explicit form of DCD in eq.(3), we can rewrite this, kinetic equation as

$$v^{\alpha-1}\frac{dv}{dt} \quad = \quad -\lambda v. \tag{4}$$

Note the possible equivalence with equation (26) in Ref. [1], by changing variables.

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The solution of this equation can be found by simple integration and is given in terms of $q - exponential$ as power-law velocity distribution,

$$v(t) \quad = \quad v_0 \left[1 - (\alpha - 1)\frac{\lambda t}{v_0^{\alpha-1}}\right]^{\frac{1}{\alpha-1}}, \tag{5}$$

that is Haff's-like law for granular gases. See and compare also eq.(1) in Ref. [8] and the eq.(27) in Ref. [1].

Another example of other applications of this kind of distribution in granular systems can be found Ref. [3], where the author claim that bubbling fluidized beds are very well fitted by a probability density function with power-law tails, following non-Gaussian statistics, typically used in the context of the Tsallis statistics.

Precedents of q-exponential appearance and similar non-Gaussian distributions can be accessed in some article and show the consistence of the approach, e.g., using the stochastic approach. An attempt to obtain velocity probability distribution function $P(v)$ for a model system of granular gas within the framework of generalized statistical mechanics, can be found in Ref. [4]. Also, in Ref. [2], the authors explore the origins of the macroscopic friction in confined granular materials using a q-Gaussian distribution, based on same non-Gaussian statistics and using porous media equation. In our recent work [6], using a variational approach with DCD, we have obtained the porous medium equation and presented insight for the solution in terms of the q-Gaussian.

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