Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 12, n. 1, 2026.

Trabalho apresentado no XLIV CNMAC, Fundação Getulio Vargas - Rio de Janeiro - RJ, 2025.

Advancing Multiphase Reservoir Simulation: Physics-Guided Expansion from Two-Phase to Three-Phase Datasets

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Physics-Guided Data Augmentation (PGDA) refers to a data augmentation approach that integrates physical laws, constraints, or properties of a system to generate new training samples in a machine learning model. In [3] PGDA was combined with unsupervised learning and improves the accuracy and stability of a bit wear machine learning model in drilling applications. In [2], a PGDA method is introduced to enhance the accuracy and generalization of neural operator models, leveraging the physical properties of differential equations. Here, we introduce a physics-guided gas estimator to expand our two-phase dataset to a three-phase dataset.

In a numerical reservoir simulation, the reservoir domain is discretized into a grid, and at each grid cell, the simulation computes the evolution of pressure and fluid saturations over time. This dynamic behavior is governed by conservation laws, fluid flow equations, and rock-fluid interactions. The governing nonlinear flow equation for a two-dimensional reservoir simulator is represented in Equation 1, where subscript j refers to the fluid phase. In a two-phase simulation, j corresponds to water and oil, while in a three-phase simulation, it includes gas. Here, k is the absolute permeability, k_r is the relative permeability, ρ is the fluid density, μ is the viscosity, and ϕ is the porosity.

$$\nabla\left[\mathbf{k}\frac{k_{r,j}(S_j)}{\mu_j}\rho_j\nabla P_j\right]+q_j-\frac{\partial}{\partial t}(\phi S_j\rho_j)=0 \tag{1}$$

The gas phase can either be dissolved in oil or exist as a separate gas phase, depending on pressure conditions. We decided to estimate gas saturation and production rate based on solubility ratios, the parameter that expresses the amount of gas present in the liquid.

Formally, gas saturation, denoted as S_g, represents the fraction of the pore volume occupied by free gas, so we can write the general equation for the flow rate of dissolved and separated gas:

$$q_g = R_s q_o \tag{2}$$

where q_g is the gas flow rate, q_o is the oil flow rate and R_s is the solubility ratio, which quantifies the amount of gas dissolved in oil at a given pressure. To properly define S_g, we carefully avoid using $S_g = 1 - S_w - S_o$, since our data comes from a two-phase reservoir where originally $S_w + S_o = 1$. Therefore, we express oil saturation as:

$$S_o = 1 - S_w \tag{3}$$

To estimate the amount of gas present per unit pore volume in the oil-filled region, we assume that the gas content is directly proportional to the solubility ratio R_s. When the pressure decreases

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due to reservoir exploration, the reservoir fluid cannot keep all gas dissolved. Consequently, some gas is released from the oil, forming a free gas phase ($S_g > 0$) To address this, we define S_g as:

$$S_g = R_s(1 - S_w) - \alpha P, \tag{4}$$

where P is the reservoir pressure and α is a small proportionality constant. This formulation ensures that at high pressure, $S_g \to 0$ meaning almost all gas remains dissolved in oil, and, at lower pressure, $S_g > 0$, meaning gas is liberated as a free phase. Moreover, following this framework, R_s follows the function:

$$R_s = R_s^{max}(1 - e^{-\gamma P}) \tag{5}$$

where, R_s^{max} is the maximum possible solubility ratio, γ is a small positive parameter that controls the growth rate of R_s and P is pressure. This formulation is physically consistent because it is exponential and monotonic, meaning R_s always increases with pressure. At low pressures ($P \to 0$), $R_s \approx 0$, ensuring that no gas dissolves in the absence of sufficient pressure. Converse0.03ly, at high pressures ($P \to \infty$), R_s approaches its upper bound $R_s^{\max}$, preventing unbounded growth. The exponential term $e^{-\gamma P}$ guarantees that R_s remains strictly non-decreasing while ensuring that solubility does not increase indefinitely. Thus, to introduce variability and reflect more realistic reservoir conditions, we decided to add Gaussian noise to gas saturation and gas production rate, using a normally distributed random variable with mean 0 and standard deviation that controls the noise level.

Following this framework, we successfully expanded a two-phase (oil and water) dataset with 300 simulations from [1] to a three-phase dataset (oil, water, and gas). Figure 1 shows a simulation with oil and water saturation and the estimated gas saturations for timesteps 0, 10, and 20.

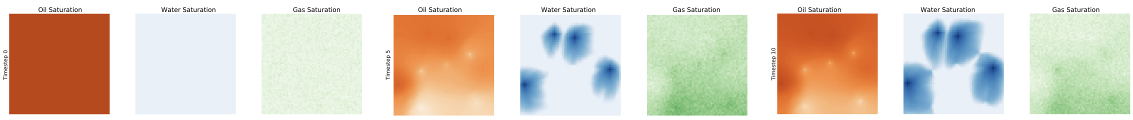

Figure 1: Water, oil and gas saturation for simulation 1. Source: elaborated by the author.

Acknowledgment

This work was carried out with the support of the Coordination for the Improvement of Higher Education Personnel (CAPES) – Brazil – Funding Code 001, the National Council for Scientific and Technological Development (CNPq), and Petrobras.

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