Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 12, n. 1, 2026.

Trabalho apresentado no XLIV CNMAC, Fundação Getulio Vargas - Rio de Janeiro - RJ, 2025.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

ANN Estimations of the Absorption Coefficient in Multi-Region Heterogenous Media: MoC Solutions as Training Data

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The estimation of the medium absorption coefficient from external measurements can be stated as an inverse problem [5, 6], and has important applications in optical medicine [8], including in optical tomography [2]. In this work, we propose a framework based on artificial neural networks (ANNs) to estimate the absorption coefficient in multi-region heterogeneous media. The associated direct transport problem [4] is given as

$$-1<\mu<1, \mu \neq 0: \frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma_t I(t,\mu,x) = \sigma_s \Psi(t,x),\ (t,x)\in(0,t_f]\times(a,b), \tag{1a}$$

$$-1<\mu<1: I(0,\mu,x)=0, x\in[a,b], \tag{1b}$$

$$\mu>0: I(t,\mu,x)=q(t,\mu),\ t\in[0,t_f], \tag{1c}$$

$$\mu<0: I(t,\mu,x)=0,\ t\in[0,t_f], \tag{1d}$$

where $I(t,\mu,x)$ $[W/sr]$ is the particle intensity at the time t $[ps]$, in the direction μ $[sr]$, and at the point x $[cm]$, c $[cm/ps]$ is the average speed of light in the medium, $\sigma_t(x) = \kappa(x) + \sigma_s(x)$ $[1/cm]$ is the total absorption coefficient, $\kappa(x)$ $[1/cm]$ is the absorption coefficient, and $\sigma_s(x)$ $[1/cm]$ is the scattering coefficient. The average scalar flux is denoted by $\Psi(t,x) = \frac{1}{2}\int_{-1}^{1} I(t,\mu',x)\,d\mu'$. Based on the model given in [1], the only source is a laser pulse given by

$$q(t,\mu) = w\left(\frac{|\mu-\mu_s|}{\delta_\mu}\right) w\left(\frac{|t-\tau_s-\delta_t|}{\delta_t}\right), \tag{2}$$

where μ_s is the laser direction, δ_μ its angular spread, τ_s its activation time, δ_t its temporally center, and $w(\nu)$ is the window function

$$w(\nu) = \begin{cases} 1 & ,\nu=0, \\ \exp\left(\left(2e^{-1/|\nu|}\right)/(|\nu|-1)\right) & ,0<\nu<1, \\ 0 & ,|\nu|\geq 1. \end{cases} \tag{3}$$

The objective is to estimate the absorption coefficient $\kappa(x)$ from detector measurements $d_0(t) = \Psi(t,a)$ and $d_1(t) = \Psi(t,b)$, $t\in[0,t_f]$. We propose to estimate κ as a piece-wise constant function. The medium is partitioned into n_c cells, which determines the resolution of the estimations. A multi-layer perceptron (MLP) neural network [3] is built to give the $\boldsymbol{\kappa} = (\kappa_i)_{i=1}^{n_c}$ estimations from discrete detectors measurements $\boldsymbol{d} = \{(d_0(t_j), d_1(t_j))\}_{j=1}^{n_d}$, where n_d is the number of measurements in discrete times. The ANN is trained from a data set $\left\{\left(\boldsymbol{d}^{(s)}, \boldsymbol{\kappa}^{(s)}\right)\right\}_{s=1}^{n_k}$ computed from solutions

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of the direct problem (1). The direct solver is based on the Method of Characteristics (MoC), and it has already been detailed in our previous work [7]. Therefore, the framework couples an efficient direct solver with a general-purpose non-linear regression model.

As a work in progress, we present here a preliminary test case. The medium is assumed to have the total absorption coefficient $\sigma_t = 1$, $a = 0$, $b = 1$, and the parameters of the laser pulse are $\mu_s = 1.0$, $\delta_\mu = 1$, $\tau_s = 0.0$, $\delta_t = 120$. The direct solver has been used to produce a training set $\left\{\left(\boldsymbol{d}^{(s)}, \boldsymbol{\kappa}^{(s)}(x)\right)\right\}_{s=1}^{n_\mathrm{k}}$ with $\boldsymbol{d}^{(s)} = \{(d_0(t_j), d_1(t_j))\}_{j=1}^{n_d}$, $t_j = 10j$, $n_d = 6$ and each output vector $\boldsymbol{\kappa}^{(s)}(x)$ contains $n_c = 10$ piecewise constant absorption coefficients distributed over the domain, defined as

$$\boldsymbol{\kappa}^{(s)}(x) = \left(\kappa_1^{(s)}, \kappa_2^{(s)}, \ldots, \kappa_i^{(s)}\right), \tag{4}$$

with $\kappa_i^{(s)} = 0.1 + (s-1)0.1$, $s = 1, 2, \ldots, n_\mathrm{c} = 10$.

Further work will include the training of the ANN and the evaluation of the estimations for different resolution setups. The framework is expected to be a powerful alternative for estimating the absorption coefficient in multi-region heterogeneous media.

Acknowledgments

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

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