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# Exact Markov Chains Monte Carlo Methods for the Normalised Power Prior

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In statistical modelling with historical data, a central challenge is effectively integrating information from past studies with current data. This is particularly relevant in clinical trials and medical research, where leveraging historical data can improve the efficiency and robustness of statistical inferences. A natural approach in this setting is the use of *informative priors*, which incorporate prior knowledge into the analysis, thereby enhancing parameter estimation and decision-making.

Among the various methods for constructing informative priors, *power priors* [5] have gained significant attention. They allow for controlled borrowing of information from historical data by adjusting a power parameter, $\eta$, which typically ranges between 0 and 1. When $\eta = 0$, no historical information is incorporated, whereas $\eta = 1$ fully integrates the historical data into the current analysis.

Despite their advantages, the posterior distribution in these models is often *doubly intractable*, making standard Markov Chain Monte Carlo (MCMC) methods challenging to apply. Current approaches rely on *approximate methods* [1, 3], which lack theoretical guarantees and explicit convergence bounds. This work addresses this gap by developing an *exact MCMC algorithm* [2, 6] capable of efficiently sampling from these complex posterior distributions.

Consider the following Bayesian schema:

$$p(\theta \mid D) = \frac{L(D \mid \theta)\pi_0(\theta)}{\int_\Theta L(D \mid t)\pi(t)\,\mathrm{d}t} = \frac{L(D \mid \theta) \overbrace{\pi_0(\theta)}^{\text{Initial prior}}}{m(D)}.$$

**Definition** (Power prior). *Let $D_0 = \{d_{01}, d_{02}, \ldots, d_{0N_0}\}$, $d_{0i} \subseteq \mathcal{X}^p$ be the historical data and let $L(D_0 \mid \theta)$ be a likelihood function assumed to be finite for all $\theta \in \Theta \subseteq \mathbb{R}^q$. Then, the power prior is defined as*

$$\pi_\eta(\theta \mid D_0) \propto L(D_0 \mid \theta)^\eta \pi_0(\theta),$$

*where $\pi_0(\theta)$ is the initial prior distribution of $\theta$ and $\eta$ is the (fixed) power parameter.*

Then, when we want to accommodate the uncertainty in the choice of $\eta$, we treat it as a random variable and assign a prior distribution to it, indexed by $\varphi$. In order to compute the posterior distribution of $\theta$ and $\eta$, we need to find the normalizing constant (from ()) $c_0(\eta)$, which is given by

$$c_0(\eta) = \int_\Theta L(D_0 \mid t)^\eta \,\mathrm{d}P_0(t),$$

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and the marginal posterior distribution of $\eta$ as, for a new dataset $D$,

Notice that, since both $c_0(\eta)$ and $\int_\Theta L(D \mid t)L(D_0 \mid t)^\eta \, \mathrm{d}P_0(t)$ are intractable, we call this a doubly-intractable posterior. Previous work [1, 3] has focused on emulating $\log c_0(\eta)$ and $\log c'(\eta) = \mathbb{E}_{\pi_\eta}[\log L(D_0 \mid \theta)]$ through non-parametric models, which has proven to be more effective than common interpolation methods—particularly when incorporating shape constraints.

Sampling from these posteriors is challenging due to the intractability of standard Metropolis–Hastings acceptance probabilities, making approximate methods like auxiliary variable techniques computationally expensive and lacking convergence guarantees; an alternative is unbiased event constructions, which enable exact sampling without evaluating normalizing constants.

For instance, applying Barker's algorithm for MCMC [7] to sample from a target distribution $\pi(x) \propto \pi'(x)$ with proposal density $q(x, y)$ has become feasible through Bernoulli factory methods. These methods enable the construction of events with probability $\alpha_B(x, y)$ when

$$\alpha_B(x,y) = \frac{\pi(y)q(y,x)}{\pi(x)q(x,y) + \pi(y)q(y,x)} = \frac{\pi'(y)q(y,x)}{\pi'(x)q(x,y) + \pi'(y)q(y,x)}$$

cannot be directly evaluated [4].

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