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On the AVD-Total Chromatic Number of 4-Regular Circulant Graphs

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Abstract. An *AVD-k-total coloring* of a simple graph G is a mapping $\pi : V(G) \cup E(G) \to \{1, \ldots, k\}$, with $k \geq 1$ such that: for each pair of adjacent or incident elements $x, y \in V(G) \cup E(G)$, $\pi(x) \neq \pi(y)$; and for each pair of adjacent vertices $x, y \in V(G)$, sets $\{\pi(x)\} \cup \{\pi(xv) \mid xv \in E(G) \text{ and } v \in V(G)\}$ and $\{\pi(y)\} \cup \{\pi(yv) \mid yv \in E(G) \text{ and } v \in V(G)\}$ are distinct. The *AVD-total chromatic number*, denoted by $\chi''_a(G)$ is the smallest k for which G admits an AVD-k-total-coloring. In 2005, Zhang et al. conjectured that any graph G has $\chi''_a(G) \leq \Delta + 3$, where Δ is the maximum degree of G and this conjecture is known as AVD-Total Coloring Conjecture (AVD-TCC). In this article, we determine that the AVD-total chromatic number of two infinite families of 4-regular circulant graphs, we prove that $\chi''_a(C_n(a, b)) = 6$, where n is even and a, b are both odd such that $1 \leq a < b < \lfloor n/2 \rfloor$, and $\chi''_a(C_n(1, k)) = 6$, where n and $\ell = \frac{n}{\gcd(n,k)}$ are even.

Palavras-chave. Total Coloring, Adjacent-Vertex-Distinguishing, Circulant Graphs

1 Introduction

Let $G = (V, E)$ be a simple connected graph and Δ be the maximum degree of G, for most definitions and notations in Graph Theory we consider [4]. A *k-total coloring* of G is an assignment of k colors to the vertices and edges of G so that adjacent or incident elements have different colors. The *total chromatic number* of G, denoted by $\chi''(G)$, is the smallest k for which G has a k-total coloring. Clearly, $\chi''(G) \geq \Delta + 1$. The Total Coloring Conjecture (TCC) states that the total chromatic number of any graph is at most $\Delta + 2$ [3, 22]. Graphs with $\chi''(G) = \Delta + 1$ are called *Type 1*, and graphs with $\chi''(G) = \Delta + 2$ are called *Type 2*. It is known that determining the total chromatic number is a NP-complete problems even for regular bipartite graphs [17].

Let π be a k-total coloring of G and let $C_\pi(u) := \{\pi(u)\} \cup \{\pi(uv) \mid uv \in E(G), v \in V(G)\}$ be the set of colors that *occur* in a vertex $u \in V(G)$. If it is clear from the context that π is a k-total coloring of G, then $C_\pi(u)$ is written simply as $C(u)$. Two vertices u and v are *distinguishable* when $C(u) \neq C(v)$. If this property is true for every pair of adjacent vertices, then π is an *Adjacent-Vertex-Distinguishing-k-Total-Coloring*, or simply *AVD-k-total coloring*. The *AVD-total chromatic number* of G, denoted by $\chi''_a(G)$, is the smallest k for which G admits an AVD-k-total coloring. If AVD-TCC holds, then we can classify any graph according to the AVD-total chromatic number. If $\chi''_a(G) = \Delta + 1$, then G is called *AVD-Type 1*. If $\chi''_a(G) = \Delta + 2$, then G is called *AVD-Type 2*. If $\chi''_a(G) = \Delta + 3$, then G is called *AVD-Type 3*. It is straightforward that any *AVD-Type 1* graph is a *Type 1* graph. However, the converse is not true. For instance, the complete graph K_n with n odd, is *Type 1* and *AVD-Type 3* [9, 23].

In 2005, Zhang et al. [24] introduced the AVD-total coloring problem. The authors determined the AVD-total chromatic number for some families of simple graphs and noted that all of them

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admit an AVD-k-total coloring with k is at most $\Delta + 3$. Based on these results, the authors posed the Conjecture 1.1, known as AVD-Total Coloring Conjecture (AVD-TCC).

Conjecture 1.1 (Zhang et al. [24]). *If G is a simple graph, then $\chi''_a(G) \leq \Delta + 3$.*

Since the proposal of this conjecture, several studies have been conducted. In 2008, Chen [8] determined that graphs with maximum degree 3 satisfy the AVD-TCC. In the same year, Chen [9] determined the AVD-total chromatic number of some graph classes with maximum degree of at least 6. In 2009, Chen and Guo [7] determined the AVD-total chromatic number of hypergraphs Q_n. In the same year, Hulgan [14] presented concise proofs of AVD-total chromatic number of cycles, complete graphs and the upper bound of graphs with maximum degree 3. Furthermore, recent studies have been conducted involving some other graph classes in order to investigate Conjecture 1.1, such as equipartite graphs, split graphs, corona graphs and 4-regular graphs [16, 19–21]. We remark that, in 2014, Papaioannou and Raftopoulou proved that AVD-TCC holds for 4-regular graphs.

A *circulant graph* $C_n(d_1, d_2, \ldots, d_l)$ with integers numbers $1 \leq d_i \leq \lfloor n/2 \rfloor$, where $1 \leq i \leq l$ has vertex set $V = \{v_0, v_1, \ldots, v_{n-1}\}$ and edge set $E = \bigcup_{i=1}^{l} E_i$, where $E_i = \{e_0^i, e_1^i, \ldots, e_{n-1}^i\}$ and $e_j^i = v_j v_{j+d_i}$ (if n is even and $d_l = n/2$, then $E_l = \{e_0^l, e_1^l, \ldots, e_{\frac{n-2}{2}}^l\}$), where the indexes of the vertices are considered modulo n.

It is known that cycle graphs $C_n \simeq C_n(1)$ are *Type 1* if and only if n is divisible by 3 [23] and are *AVD-Type 2*, if $n \neq 3$ and *AVD-Type 3*, if $n = 3$ [24]. It is well known that the connected cubic circulant graph is written as $C_{2n}(d, n)$, such that n and d are positive integers such that $1 \leq d < n$ and $n \geq 2$. In 2004, the total chromatic number of every cubic circulant graph was determined by Hackmann and Kemnitz [12]. Recently, it was proved that every cubic circulant graph is *AVD-Type 2* [1].

Some partial results are known on power of cycle graphs, the infinite family of circulant graphs $C_n^k := C_n(1, 2, \ldots, k)$ with $2 \leq k \leq \lfloor n/2 \rfloor$. In 2003, Campos and de Mello [6], proved that $C_n(1,2)$ is *Type 1*, except for graph $C_7(1,2)$ which is *Type 2*, and they conjectured that C_n^k, with $2 \leq k \leq \lfloor n/2 \rfloor$, is *Type 2* if and only if n is odd and $k > \frac{n}{3} - 1$ [5]. In 2019, Zorzi [25] proved that this conjecture holds for $k = 3$ and $k = 4$ and in the same year, Zorzi et al. [26] proved that power of cycle graphs with k even and $n \geq 4k^2 + 2k$ are all *Type 1*. In 2021, Geetha et al. [11] proved that some infinite families of power of cycle graphs verify Campos and de Melo's conjecture. In 2019, Alvarado et al. [2], on AVD-total chromatic number of power of cycles, proved that C_n^2 and C_n^k with $n \equiv 0 \mod (k+1)$ are *AVD-Type 2*. For 4-regular circulant graphs, some infinite families have their total chromatic number determined [10, 15, 18]. On AVD-total chromatic number of 4-regular circulant graphs few results are known: the complete graph K_5 is the only 4-regular circulant graph known which is *AVD-Type 3*; and C_n^2 is *AVD-Type 2* (see Figure 1).

In this paper, we prove that the 4-regular circulant graphs $C_n(a, b)$ are *AVD-Type 2*, where n is even, a and b are odd such that $1 \leq a < b < \lfloor n/2 \rfloor$, and $C_n(1, k)$ are *AVD-Type 2* where n and $\ell = \frac{n}{\gcd(n,k)}$ are even.

2 Main Result

In this section, we prove that all members of two infinite families of 4-regular graphs are *AVD-Type 2*. Specifically, we prove that $C_n(a, b)$ is *AVD-Type 2*, if n is even and a and b are both odd such that $1 \leq a < b < \lfloor n/2 \rfloor$ (see Corollary 2.1); and $C_n(1, k)$ is *AVD-Type 2* (see Theorem 2.1), where n and $\ell = \frac{n}{\gcd(n,k)}$ are even. To prove both results, we use two auxiliary properties that allow us to determine the AVD-total chromatic number of these graph classes.

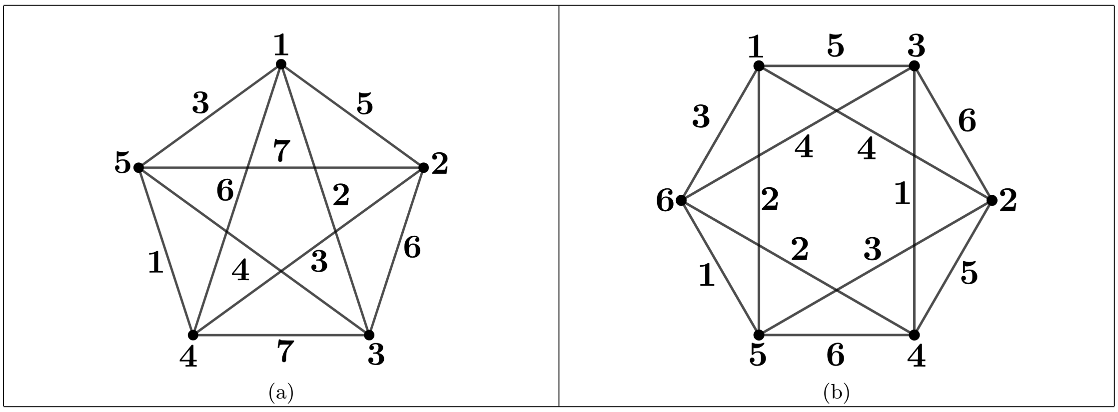

Figure 1: Examples of circulant graphs such that their AVD-total chromatic number were determined. In Figure 1a, the complete graph $K_5 \simeq C_5(1, 2)$ which is the only known 4-regular *AVD-Type 3* graph. In Figure 1b, the power of cycle graph $C_6^2 \simeq C_6(1, 2)$ which is *AVD-Type 2*. Source: the authors.

Proposition 2.1 (Zhang et al. [24]). *Let G be a simple graph. If G has two adjacent vertices of maximum degree, then $\chi_a''(G) \geq \Delta + 2$. On the other hand, if G does not have adjacent vertices of maximum degree, then $\chi_a''(G) \geq \Delta + 1$.*

Proposition 2.2 (Chen [9]). *If G is a bipartite graph, then $\chi_a''(G) \leq \Delta + 2$.*

The following lemma characterizes the bipartite circulant graphs (Lemma 2.1).

Lemma 2.1 (Heuberger [13]). *The circulant graph $C_n(d_1, d_2, \ldots, d_l)$ is bipartite if and only if n is even and d_i is odd, for all $i \in \{1, 2, \ldots, l\}$.*

Corollary 2.1. *The 4-regular circulant graph $C_n(a, b)$, with n even, and such that a and b are both odd, is AVD-Type 2.*

Proof. Since n is even and a and b are odd, from Lemma 2.1, $C_n(a, b)$ is bipartite. From Proposition 2.1, $\chi_a''(C_n(a, b)) \geq 6$ and from Proposition 2.2, $\chi_a''(C_n(a, b)) \leq 6$. Therefore, $\chi_a''(C_n(a, b)) = 6$, i.e., $C_n(a, b)$ is *AVD-Type 2*. ☐

Observe that, according to Corollary 2.1, the 4-regular circulant graph $C_n(1, k)$ is *AVD-Type 2* when n is even and k is odd. The remaining case for even n occurs when k is even. Theorem 2.1 provides a partial result for this case, assuming that $\ell = \frac{n}{\gcd(n,k)}$ is even.

Theorem 2.1. *Let n and k even positive integers such that $1 < k < n/2\rfloor$. If $\ell = \frac{n}{\gcd(n,k)}$ is even, then 4-regular circulant graph $C_n(1, k)$ is AVD-Type 2.*

Proof. We ask the reader to refer to Figures 2 and 3 to understand the coloring procedure defined below. Let $G := C_n(1, k)$ be the 4-regular graph such that $1 < k < \lfloor n/2 \rfloor$ and n even. Since $C_n(1, k)$ has two adjacent vertices of maximum degree, from Proposition 2.1, $\chi_a''(C_n(1, k)) \geq 6$. In order to prove that $\chi_a''(C_n(1, k)) = 6$ we depict an AVD-6-total coloring π of $C_n(1, k)$. We remark that $E(C_n(1, k)) = E_1 \cup E_2$ such that $E_i = \{e_j^i \mid j \in \{0, 1, \ldots, n-1\}\}$, where $e_j^1 = v_j v_{j+1}$ and $e_j^2 = v_j v_{j+k}$. So, we can partite the $C_n(1, k)$ into two subgraphs, G_1 which is induced by E_1 and G_2 which is induced by E_2. We remark that G_1 is one cycle with lenght n and G_2 is a disjoint union of $\gcd(n,k)$ cycles with lenght $\ell = \frac{n}{\gcd(n,k)}$. Therefore, we set:

$$G_2 := \bigcup_{p=0}^{\gcd(n,k)-1} \mathcal{C}_p,$$

such that, $p \in \{0, 1, \ldots, \gcd(n,k)-1\}$. Notice that $\mathcal{C}_p$ is the induced subgraph $G[S_p]$ by the set $S_p = \{v_{p+jk} \mid j \in \{0, 1, \ldots, \ell-1\}\}$ and we set $w_j^p := v_{p+jk}$. Figure 2 depicts the subgraph G_2 of $C_{16}(1,4)$ as four cycles $\mathcal{C}_0$, $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$.

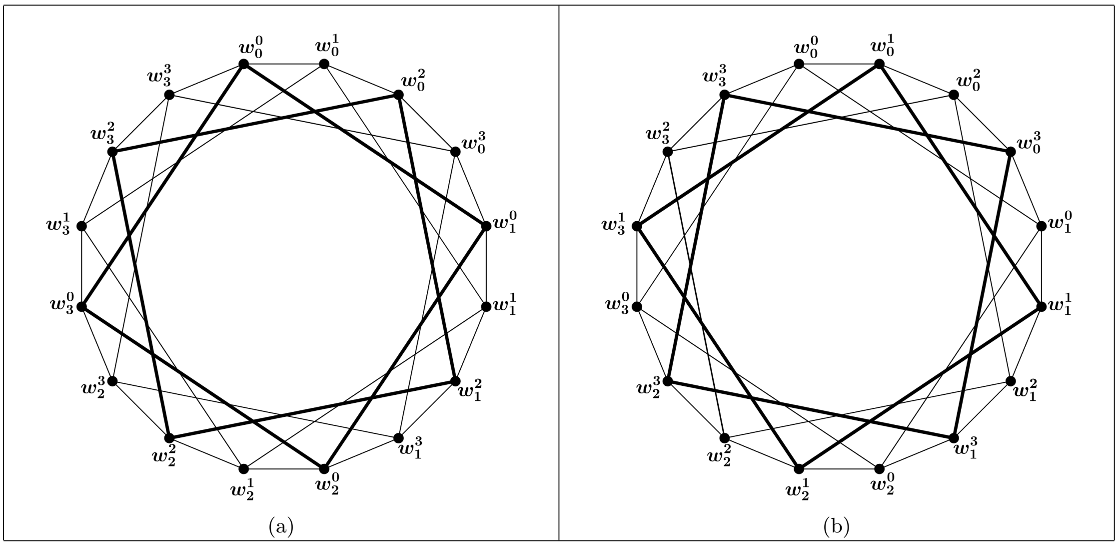

Figure 2: The circulant graph $C_{16}(1,4)$. Since $\gcd(16,4) = 4$, G_2 is a disjoint union of four cycles $\mathcal{C}_0$, $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$. In Figure 2a, we depict the cycles $\mathcal{C}_0$ and $\mathcal{C}_2$ with bold edges. In Figure 2b, we depict the cycles $\mathcal{C}_1$ and $\mathcal{C}_3$ with bold edges. Source: the authors.

The AVD-6-total coloring $\pi : V(G) \cup E(G) \to \{1,2,3,4,5,6\}$ is defined as follows:

For each cycle $\mathcal{C}_p$, we assign to the vertices the colors 1 and 2 alternately if p is even, and the colors 3 and 4 alternately if p is odd. We assing to the edges the colors 3 and 4 alternately if p is even, and 1 and 2 alternately if p is odd. Consider $j \in \{0, 1, \ldots, \ell-1\}$. Therefore, for p even, π is defined by:

$$\pi(w_j^p) = \begin{cases} 1, & \text{if } j \text{ is even} \\ 2, & \text{if } j \text{ is odd} \end{cases} \quad \pi(w_j^p w_{j+1}^p) = \begin{cases} 3, & \text{if } j \text{ is even} \\ 4, & \text{if } j \text{ is odd} \end{cases} \tag{1}$$

and for p odd:

$$\pi(w_j^p) = \begin{cases} 3, & \text{if } j \text{ is even} \\ 4, & \text{if } j \text{ is odd} \end{cases} \quad \pi(w_j^p w_{j+1}^p) = \begin{cases} 1, & \text{if } j \text{ is even} \\ 2, & \text{if } j \text{ is odd} \end{cases} \tag{2}$$

For edges $v_i v_{i+1} \in E_1$ with $i \in \{0, 1, \ldots, n-1\}$, we assign the colors 5 and 6 alternately:

$$\pi(v_i v_{i+1}) = \begin{cases} 5, & \text{if } i \text{ is even} \\ 6, & \text{if } i \text{ is odd} \end{cases} \tag{3}$$

In order to prove that π is an AVD-6-total coloring we will show that:

1. π is a 6-total coloring. Since k is even, every index vertex i of v_i has the same parity as index vertex $i+k$ of v_{i+k}. Therefore, the adjacent vertices v_i and v_{i+k} belongs to the same cycle $\mathcal{C}_p$. Since ℓ is even, the alternate colors given in (1) and (2) form a 4-total coloring of the cycles $\mathcal{C}_p$. Since n is even, G_1 is an even cycle. Therefore, alternate colors 5 and 6 given in (3) induces a 6-total coloring of G_1, where vertices with even indexes are assign by 1 or 2 and odd indexes are assign by 3 or 4. Therefore π is a 6-total coloring.

2. each pair of adjacent vertices are distinguishable. Suppose that i is even, then from (1), v_i and v_{i+k} are assigned by 1 or 2. If $\pi(v_i) = 1$, then $\pi(v_{i+k}) = 2$ and $2 \notin C(v_i)$. The same argument can be done supposing that $\pi(v_i) = 2$. Therefore, v_i and v_{i+k} are distinguishable, i.e., $C(v_i) \neq C(v_{i+k})$. Suppose that i is odd, then from (2), v_i and v_{i+k} are assigned by 3 and 4. If $\pi(v_i) = 3$, then $\pi(v_{i+k}) = 4$ and $4 \notin C(v_i)$. The same argument can be done supposing that $\pi(v_i) = 4$. Therefore, v_i and v_{i+k} are distinguishable, i.e., $C(v_i) \neq C(v_{i+k})$. Finally, we prove that v_i and v_{i+1} are distinguishable. Suppose that i is even. Since there are incident edges of v_i that are assigned with colors 3 and 4, $\{3,4\} \subset C(v_i)$. Therefore, if $\pi(v_{i+1}) = 3$ implies that $4 \notin C(v_{i+1})$. Hence, $C(v_i) \neq C(v_{i+1})$. Suppose that i is odd. Since there are incident edges of v_i that are assigned with colors 1 and 2, $\{1,2\} \subset C(v_i)$. Therefore, if $\pi(v_{i+1}) = 1$ implies that $2 \notin C(v_{i+1})$. Hence, $C(v_i) \neq C(v_{i+1})$. Thus, v_i and v_{i+1} are distinguishable.

Thus, π is an AVD-6-total coloring of $C_n(1,k)$. Figure 3 depicts the circulant graph $C_{16}(1,4)$ with an AVD-6-total coloring. □

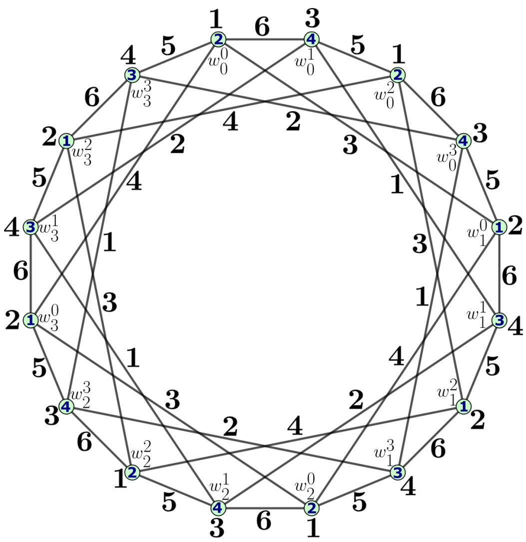

Figure 3: The circulant graph $C_{16}(1,4)$ with the AVD-6-total coloring π of $C_{16}(1,4)$. The vertices of $C_{16}(1,4)$ are labelled with set $w_j^p := v_{p+jk}$. The blue assignment in the internal region of vertex $u \in V(C_{16}(1,4))$ is the color that does not belong to the set $C(u)$. Source: the authors.

3 Concluding Remarks

In this work, we determine the AVD-total chromatic number for two infinite families of 4-regular circulant graphs. We prove that $\chi''_a(C_n(a,b)) = 6$ when n is even and a, b are odd integers such that $1 \leq a < b < \lfloor n/2 \rfloor$, and that $\chi''_a(C_n(1,k)) = 6$ when both n and $\ell = \frac{n}{\gcd(n,k)}$ are even. These results show that all graphs in these families are *AVD-Type 2* and provide further evidence supporting the AVD-Total Coloring Conjecture.

For the 4-regular circulant graph $C_n(1,k)$, the remaining open cases include instances where n is even, k is even, and ℓ is odd. These cases require a distinct coloring strategy, as the auxiliary decomposition used in Theorem 2.1 depends on the evenness of ℓ. Furthermore, the case when n is odd remains entirely open and poses additional combinatorial challenges, particularly due to structural differences in the decomposition of circulant graphs of odd order.

Given that the only known 4-regular circulant graph that is *AVD-Type 3* is $C_5(1,2)$, and based on the current state of the art and the techniques developed in this work, we propose Conjecture 3.1.

Conjecture 3.1. *Let n and k be positive integers such that $1 < k \leq \lfloor n/2 \rfloor$ and $n \geq 5$. Every 4-regular circulant graph $C_n(1,k)$ is AVD-Type 2, except for $C_5(1,2)$ which is AVD-Type 3.*

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