

## A mathematical study of the viability of the polymer injection process: a simplified model.

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**Abstract.** It is well known that adding some water soluble polymer into water can enhance the oil recovery factor in the waterflood recovery method. However, this polymer solution increases the injection cost. Not only in reason of the polymer cost itself, but also because of the time for production should be longer in order to reach the water breakthrough. In this work, to study this behavior, we use a simple PDE system that models the flow of water with polymer into a one directional porous media previously filled with oil. Under suitable simplifications, we prove that the system's solution projects to a single scalar equation which we use to create a profit functional. The profit functional allows the study of the economical viability of the polymer injection. This study presents a simple new way to deal with this class of problems where we dynamically obtain regions to optimize the defined functional solution.

**Keywords.** Polymer flow in porous media, Conservation laws, Improved oil recovery

### 1 Introduction

When an oil reservoir is drilled, the oil present in the ground naturally comes out through the production well (primary recovery). However, as the pressure inside the reservoir decreases, the oil flow ceases. To keep the production wells running with oil, it is necessary to apply Enhanced Oil Recovery (EOR) methods.

Despite some methods were developed in order to enhance the oil recovery factor, only in very specific cases it is economically attractive to the companies to apply such methods in a way that optimizes the oil recovery. That means that a significant amount of the available oil is abandoned into the reservoir for economical reasons.

For any chosen technique the main question is: "Is this technique viable economically?" Usually, this question is hardly answered. In this work we propose a way to study this problem. We analyze an specific EOR method, polymer flooding, used at the oil recovery.

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## 2 The model system: two phase flow and profit optimization

The system of equations that models the two phase flow (oil and water with polymer) into an one dimensional porous media, ignoring the gravity and capillary pressure effects, is written as (see [1]):

$$\frac{\partial}{\partial t} S(x, t) + \frac{\partial}{\partial x} f(S, c) = 0, \quad (1)$$

$$\frac{\partial}{\partial t} (c(x, t)S(x, t) + \alpha(c)) + \frac{\partial}{\partial x} c(x, t)f(S, c) = 0. \quad (2)$$

Here  $c(x, t)$  is the polymer concentration in  $(x, t)$ .  $S(x, t)$  and  $f(S, c)$  are, respectively, the normalized water concentration and the fractional flux of water which are given by

$$S(x, t) = \frac{S_w(x, t) - S_{wi}}{1 - S_{or} - S_{wi}}; \quad f(S, c) = \frac{S^{n_w}}{S^{n_w} + \frac{k_{rowi}}{k_{rwor}} (1 - S)^{n_o} \frac{\mu(c)}{\mu_o}}. \quad (3)$$

Here  $S_w(x, t)$  is the water saturation at a given place and time  $(x, t)$  and  $S_{wi}$  and  $S_{or}$  are the irreducible water and the residual oil respectively.  $n_w$  and  $n_o$  are non-negatives constants.  $k_{rowi}$  and  $k_{rwor}$  are constants related to the relative permeability to water and oil respectively.  $\mu_o$  is the oil viscosity.  $\mu(c)$  is the water and polymer solution viscosity given by  $\mu(c) = \mu_w + mc^\lambda$ , where  $\mu_w$  is the water viscosity and  $m$  and  $\lambda$  are constant parameters depending on the chosen polymer. Generally, most of these parameters are obtained empirically.

Once the Riemann problem for this PDE system is solved, what we do analytically after some simplifications, we have access to the total time of production ( $T$ ) and to the amount of oil produced ( $Q_w$ , which is the same amount of solution - water and polymer - injected into the reservoir) with a given polymer concentration  $c$ . With these information besides data of the oil ( $P_o$ ), polymer ( $P_{pol}$ ), water ( $P_w$ ), and time ( $P_t$ ) prices and a fix cost ( $k$ ) we use the functional below to compute the profits of production for a given polymer concentration ( $\mathcal{L}(c)$ ).

$$\mathcal{L}(c) = (P_o - P_w)Q_w(c) - (P_{pol} - P_w)cQ_w(c) - P_t T(c) - k. \quad (4)$$

We are interested in maximizing this functional. For the most general case, it is possible to find the maximum profit numerically. Hereafter, with some restrictions, it is possible to find, analytically, good properties and behavior patterns. Both numerical and analytical methods which we use to approach the functional are meticulous developed in this work.

## References

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