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Flow of Brine Solution with Dissolved Ions in an 1-Directional Porous Media

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Abstract. In this work we consider the injection of water with dissolved ions into a linear horizontal porous rock cylinder with constant porosity and absolute permeability initially containing oil and water in several proportions. The water is assumed to have low salinity concentration, where some ions are dissolved, we also assume that there is in the rocks some possible minerals that can dissolve or precipitate in water phase. There are two chemical fluid components as well as two immiscible phases: water and oil, (w, o) . The dissolved ions are: positive divalent ions: calcium ions, Ca^{2+} and magnesium ions, Mg^{2+} ; negative divalent ions: sulphate ions, SO_4^{2-} ; positive monovalent ions: sodium ions, Na^+ ; negative monovalent ions: chloride ions, Cl^- . The cations are modeled to be involved in fast ion exchange process with a surface negative S^- which can absorb the positive ions, Ca^{2+} , Mg^{2+} and Na^+ ; the quantities of cations absorbed for the surface S^- are denoted by β_{Ca} , β_{Mg} and β_{Na} , for calcium, magnesium and sodium. We use simple mixing rules. We disregard any heat of precipitation/dissolution of substance reactions or ion desorption. Moreover we disregard any volume contraction effects resulting from mixing and reactions in any phase. Here we solve the Riemann problem and we give several numerical examples.

Keywords. Porous Media, Hyperbolic Equations, Riemann Problems.

1 The chemical, physical and mathematical models

The problem considered in this work is the injection of brine water into porous media, used for oil recovering. We have some chemical reactions in the model. The first one is related with the desorption of ions by the negative surface S^- , typically this surface is formed by clay. This surface is assumed to capture some positive ions (divalent and monovalent) dissolved in the water phase, see [1]. The desorption of this ions can modify the relative permeability of the rock surface. Another important phenomena into the porous media, is associated with the reactions between the mineral phases and the ion in water phase. Since we assume that we have calcite, magnesite and sulphate, we can obtain the equations of these reactions (dissolution/precipitation) in the water phase.

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Finally, we can identify the low salinity effects under the relative permeabilities of each phase. We assume that these permeabilities are modified by the surface desorption of divalent positive ions.

The complete model for transport and reaction of ions in the water phase is modelled by the following equations:

$$\phi \frac{\partial}{\partial t} C_{Ca} s_w + \frac{\partial}{\partial t} \beta_{Ca} M_c + \frac{\partial}{\partial x} C_{Ca} u_w = \frac{\partial}{\partial x} \left(\mathcal{D}(\phi, s_w) \frac{\partial}{\partial x} C_{Ca} \right) - \frac{\partial}{\partial t} \rho_c - \frac{\partial}{\partial t} \rho_g, \quad (1)$$

$$\phi \frac{\partial}{\partial t} C_{Mg} s_w + \frac{\partial}{\partial t} \beta_{Mg} M_c + \frac{\partial}{\partial x} C_{Mg} u_w = \frac{\partial}{\partial x} \left(\mathcal{D}(\phi, s_w) \frac{\partial}{\partial x} C_{Mg} \right) - \frac{\partial}{\partial t} \rho_m, \quad (2)$$

$$\phi \frac{\partial}{\partial t} C_{Na} s_w + \frac{\partial}{\partial t} \beta_{Na} M_c + \frac{\partial}{\partial x} C_{Na} u_w = \frac{\partial}{\partial x} \left(\mathcal{D}(\phi, s_w) \frac{\partial}{\partial x} C_{Na} \right), \quad (3)$$

$$\phi \frac{\partial}{\partial t} C_{SO_4} s_w + \frac{\partial}{\partial x} C_{SO_4} u_w = \frac{\partial}{\partial x} \left(\mathcal{D}(\phi, s_w) \frac{\partial}{\partial x} C_{SO_4} \right) - \frac{\partial}{\partial t} \rho_g, \quad (4)$$

$$\phi \frac{\partial}{\partial t} C_{Cl} s_w + \frac{\partial}{\partial x} C_{Cl} u_w = \frac{\partial}{\partial x} \left(\mathcal{D}(\phi, s_w) \frac{\partial}{\partial x} C_{Cl} \right), \quad (5)$$

where $\partial \rho_c / \partial x$, $\partial \rho_m / \partial x$ and $\partial \rho_g / \partial x$ are the dissolution/precipitation rates. \mathcal{D} is diffusion coefficient that depends on the water saturation and porosity and be expressed by the molecular diffusion. In system there are two saturations unknowns: water, s_w and oil, s_o ; the Darcy speed u ; one thermodynamical variables: temperature, T ; five concentrations for the dissolved ions: calcium, C_{Ca} , magnesium, C_{Mg} , sulphate, C_{SO_4} , sodium, C_{Na} and chloride, C_{Cl} are the concentrations; three concentrations for the present rock in the reservoir: calcite, ρ_c , magnesite, ρ_m , and sulphate, ρ_g . We also assume that $s_w + s_o = 1$, i.e., the pore volume fraction is filled with the liquid water and oleic phases respectively. We assume that the fluids are incompressible and that the pressure changes are so small that they do not affect the physical properties of the fluids.

In addition to equations (1)-(5) we need some algebraic equations given some equilibrium between electrical charges of some ions.

In this work, we disregard the effects of diffusion and reactions and we are able to prove that the system of equations is hyperbolic. Moreover, this class of equations introduces the formalism to a new class of problems called *partial differential algebraic equations*. Here, we solve the Riemann problem associated to the simplified problem, that are the system of equations with initial conditions which are constant by parts and is discontinuous only for $x = 0$. Moreover, we prove that the eigenvalues are positive and we utilize a upwind method for this new class of equations. We corroborate our Riemann solution with rich numerical experiments.

References

- [1] A. Omekeh, H. A. Friis, I. Fjelde and S. Evje, Modeling of Ion-Exchange and Solubility in Low Salinity Water Flooding, Society of Petroleum Engineers, SPE 154144, (2012).