IDENTIFICATION OF MECHANICAL SYSTEMS THROUGH VOLTERRA SERIES - STUDY OF BENCHMARK CASES

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Abstract— Nonlinear effects are broadly presents in several kinds of mechanical systems. Thus, it is necessary to use a suitable tool that becomes possible to characterize these nonlinearities in many situations. Volterra series can be useful for describing nonlinear systems through multiple convolutions. In this sense, the main goal of this work is to approximate the solutions of the motion equations using Volterra series in order to describe the nonlinear dynamical behavior of some mechanical benchmarks. Duffing oscillator, bilinear oscillator and a quadratically damped oscillator are analyzed to illustrate the efficiency, advantages and drawbacks of the proposed approach.

Keywords— nonlinear system identification, discrete-time Volterra series, benchmark systems

1 Introduction

System identification methods can be a good approach to estimate a model for describing the dynamical behavior. Classical linear parametric and nonparametric methods are currently used for this purpose (Aguirre, 2007; Ljung, 2007). These strategies are based on superposition principle and they are not available considering nonlinear structures. Although it may be possible to represent systems which are perturbed over a restricted operating range by a linear model, in general, nonlinear processes only can be described adequately by a nonlinear models, such as Volterra models (Billings, 1980).

The Volterra series were introduced by Vito Volterra by the end of the XIX century. These models are an alternative technique based on a functional power series known as Volterra series. Volterra series have been extensively applied in many areas such as biological systems (Zhang et al., 1998), electrical engineering in the modeling of nonlinear circuits (Bojorsell, 2008), the prediction of wind farm (Lee, 2011), signal processing involving electrodynamic loudspeaker (Kaizer, 1987), adaptive filtering (Ogunfunmi, 2007), aeroelastic systems (Silva, 2005), structural health monitoring (Caterjee, 2009), etc.

Volterra formulation have been used in the discrete time-domain involving nonlinear mechanical applications (da Silva, 2011) and in the frequency-domain through an intuitive representation known as Generalized Frequency Response Functions (GFRF's) (Peng et al., 2008). The advantage of Volterra formulation are the Volterra multidimensional kernels that represent the direct generalization of impulse response function (IRF) from the linear dynamical models and allows to separate the response of the system in linear and nonlinear components. However, this technique can present some drawbacks and is limited by several reasons (da Silva et al., 2010), numerical problems of response convergence due to a large number of samples is required to described the Volterra kernels and the identification of kernels is difficult to achieve for complex analytical models due to over-parametrization.

Fortunately, a convenient expansion of Volterra model on an orthonormal basis, known as a Wiener series, can overcome the above inconvenient and limitations (Schetzen, 1980). This idea was introduced by Norbert Wiener (Wiener, 1958) and consists in to expand the real-physics system in terms of an orthonormal basis described by functions, e.g., Kautz functions (Kautz, 1954). The representation using the Kautz basis allows to describe and identify nonlinear systems with dominant dynamic of second order because they are composed by complex conjugated poles. Thus, the orthonormal Kautz functions are more suitable in many situations involving nonlinear systems with strong oscillatory behavior. The orthonormality property of those functions characterized the fact that the orthonornal series allows to reduce drastically the parameters to be estimated and can increase the speed of convergence in problems identification (da Rosa et al., 2007).

In this sense, the aim of this work is to de-

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scribe the dynamical behaviour of three benchmarks systems: Duffing oscillator, bilinear oscillator and a quadratically damped oscillator using Volterra series expanded in the orthonormal Kautz basis. Thus, the paper is organized as follow. First of all are presented the discrete Volterra models. Next, the identification procedure using a conventional least squares approach with orthonormal Kautz basis are briefly reviewed. The identification and dynamical behaviour of the benchmarks are discussed in section 4. Finally, the results are discussed and suggestions are proposed in the section 5 for further applications.

2 Discrete Volterra series

Volterra series has been widely used for modeling and identification of nonlinear systems. In the general case for discrete-time, Volterra series relate the output $y(k)$ and the input $u(k)$ of a SISO causal system by the following expression (Schetzen, 1980):

$$y(k) = \sum_{m=1}^{+\infty} H_m(k) \tag{1}$$

in which the Volterra functional operator $H_m(k)$ is given by multidimensional convolutions:

$$H_m(k) = \sum_{n_1=0}^{N_1} \cdots \sum_{n_m=0}^{N_m} h_m(n_1, \ldots, n_m) \prod_{i=1}^{m} u(k-n_i)$$

where h_m are the m th-order Volterra kernels considering the truncated values $N_1, \ldots, N_m$ for each kernel. It is worth to note that by Eqs. (1) and (2) is the generalization of the impulse response function (IRF) for a linear dynamic systems:

$$y(k) = \sum_{n_1=0}^{N_1} h_1(n_1)u(k-n_1) \tag{2}$$

To illustrate the main idea of Volterra formulation, Figure 1 shows the schematic representation of M th-order Volterra model. Although

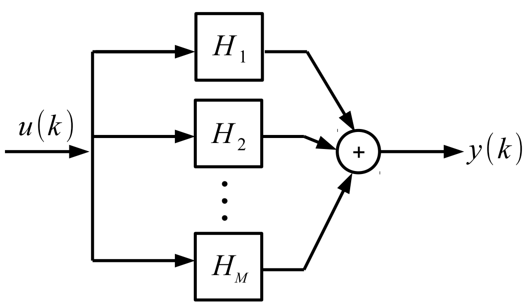

Figure 1: Block diagram representation of a system characterized by a M th-order Volterra model.

Volterra models have nice properties to be used in a broad class of non-linear identification, this formulation present drawbacks for modeling systems when the number of samples collected is high (da Silva, 2011). Fortunately, to describe the Volterra series on an orthonormal basis can be an efficient way to overcome these inconvenient. Thus, section 3 briefly reviews the identification procedure of Volterra kernels expanded in the orthonormal Kautz basis with a least square approach applied in some common mechanical systems.

3 Identification of Volterra kernels

In order to identify a given nonlinear system, classical methods and analytical techniques, such as harmonic probing method (Bedrosian and Rice, 1971; Cafferty and Tomlinson, 1997), can be used to obtain the Volterra kernels. By the least square approach, the vector with the values desired of Volterra kernels can be obtained solving:

$$\hat{\mathbf{\Theta}} = [\mathbf{U}^T\mathbf{U}]^{-1}\mathbf{U}^T\mathbf{y} \tag{3}$$

In practice, the number of parameters of the vector $\hat{\mathbf{\Theta}}$ can be large due to oscillatory systems with large memory. Increasing the number of kernels, the number of parameters to be estimated increases significantly. One way to overcome these drawbacks and over-parametrization effects is to expand the Volterra models in terms of an orthonormal Kautz basis (da Silva et al., 2010). The Kautz basis functions are very effective in representing the orthogonal kernels to identify the Volterra kernels associated with oscillatory dynamical systems (Kautz, 1954; Wahlberg, 1994). The pairs of Kautz functions $\Psi_j(z)$ has complex conjugate poles represented by $\beta_{2g-1} = \sigma + j\omega$ and $\beta_{2g} = \sigma - j\omega$ such as $|\beta_{2g-1}|,|\beta_{2g}| < 1$ for stable system. Thus, the generalized pairs of Kautz functions are given by (Heuberger et al., 2005):

$$\Psi_{2j-1}(z) = \frac{\sqrt{1-b^2}\sqrt{1-c^2}}{z^2 + b(c-1)z - c}[H_{b,c}(z)]^{j-1} \tag{4}$$

$$\Psi_{2j}(z) = \Psi_{2j-1}(z)\frac{z-b}{\sqrt{1-b^2}} \tag{5}$$

where $H_{b,c}(z) = \frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}$ and the scalar values b and c, relative to the poles β_{2g-1}, β_{2g} considered are given by:

$$b = \frac{\beta_{2g-1} + \beta_{2g}}{1 + \beta_{2g-1}\beta_{2g}} \text{ and } c = -\beta_{2g-1}\beta_{2g} \tag{6}$$

where the poles in the continuous-domain are function of parameters ω_{ng} and ξ_g described by

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$\beta_g = -\xi_g\omega_{ng} \pm j\omega_{ng}\sqrt{1-\xi_g^2}$. Optimization procedures can be used for the choice of the Kautz poles (da Rosa et al., 2007). In this work, genetic algorithm is used and based on a minimization of the prediction error function of the Volterra models using the Euclidean norm:

$$J(\rho, k) = \| y(k) - \hat{y}(\rho, k) \| \qquad (7)$$

with $\rho = \{J_1, \ldots, J_m, \xi_2, \ldots, \xi_g, \omega_2, \ldots, \omega_{ng}\}$ where $J_1, \ldots, J_m$ are the number of functions/filters and $\hat{y}(\rho, k)$ is the total output estimated using Votlerra models. After building the continuous Kautz filter using the optimal poles based on parameters ω_{ng} and ξ_g, it is necessary to use a transformation to the discrete domain $z_g = e^{\beta_g \cdot \Delta t}$, where Δt is the sampling rate. After this, assuming that the kernels $h_m(n_1, \ldots, n_m)$ in Eq. (2) are absolutely summable on $[0, \infty]$, it is possible to approximate the functional Volterra kernels by the Wiener series using the following expression:

$$H_m(k) \approx \sum_{i_1=1}^{J_1} \cdots \sum_{i_m=1}^{J_m} \alpha_{i_1,\ldots,i_m} \prod_{j=1}^{m} l_{i_j}(k) \qquad (8)$$

where $\alpha_{i_1,\ldots,i_m}$ are the coefficients used in the orthonormal basis. The term $l_{i_j}(k)$ is represented by:

$$l_{i_j}(k) = \sum_{n_i=0}^{V-1} \psi_{i_j}(n_i) u(k - n_i) \qquad (9)$$

where $V = max\{J_1, \ldots, J_m\}$. Now, rewriting Eq. (3) based on the orthonormal Kautz basis described in Eqs. (8) and (9), the estimative of vector $\mathbf{\Phi}$ composed by the coefficients $\alpha_{i_1,\ldots,i_m}$ is given by:

$$\hat{\mathbf{\Phi}} = (\mathbf{\Gamma}^T\mathbf{\Gamma})^{-1}\mathbf{\Gamma}^T\mathbf{y} \qquad (10)$$

where the matrix $\mathbf{\Gamma}$ contains the input regressors filtered by Kautz filters ψ_{i_j}.

4 Illustrative examples

Some examples are made to illustrate the approach. In each nonlinear system simulated, the output signal $y(k)$ were approximated through the Newmark method integration solved with Newton-Raphson procedure.

4.1 Duffing oscillator

The Duffing oscillator model subject to chirp input signal $u(k)$, with amplitude 0.5 [N] sweeping the frequency range from 1 to 200 [Hz], is described by nonlinear motion equation:

$$m\ddot{y}(k) + c\dot{y}(k) + k_1 y(k) + \eta(k) = u(k) \qquad (11)$$

where $m = 0.075$ [kg], $c = 0.5$ [N.s/m], $k_1 = 2.2\text{x}10^3$ [N/m] and the nonlinear term $\eta(k)$ is given by $\eta(k) = k_2 y^2(k) + k_3 y^3(k)$ where $k_2 = 4.9\text{x}10^5$ [N/m^2], $k_3 = 8.1\text{x}10^7$ [N/m^3]. A sampling frequency of 1 [kHz] and 8192 samples were used. In order to obtain the first, second and third order Volterra kernels, the first step is to build the Kautz filters from the parameters ω_{ng} [rad/s] and ξ_g with $g = 1, 2, 3$. The natural frequency of the system represented in Eq. 11 is $\omega_{n1} = 171.27$ [rad/s] and the damping factor is $\xi_1 = 0.0195$. In the optimization results, it were obtained $J_1 = 10$ and $J_2 = J_3 = 6$ Kautz functions, $\omega_{n2} = 168.32$ and $\omega_{n3} = 193.67$ [rad/s], $\xi_2 = 0.0452$ and $\xi_3 = 0.0902$ considering 50 generations, the population size equals 100 and crossover fraction equals 0.8. After this, the input was filtered by the impulse response of the Kautz functions Ψ_{i_j} and it was processed the vector Θ with the estimative for the coefficients α_{i_1,i_2,i_3} related with the respective kernel. Finally, the Volterra kernels were estimated solving Eq. 8 illustrated, respectively, in the Figures 2, 3 and 4.

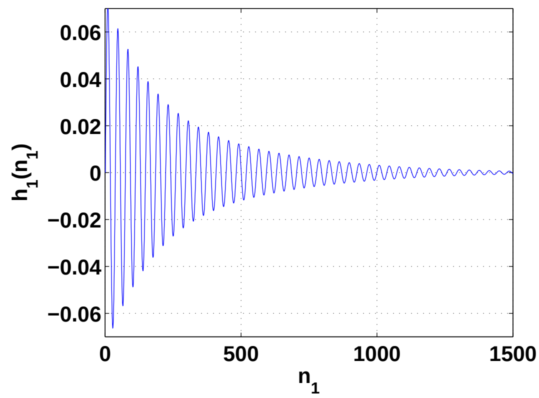

Figure 2: 1^{st} Volterra kernel.

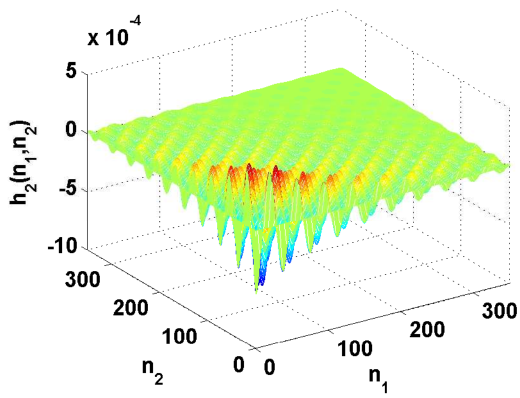

Figure 3: 2^{nd} Volterra kernel.

After the estimative of each Kernel considered in the identification procedure it was processed the total output $\hat{y}(k)$ considering the model identified. Figure 5 shows a considerable estimative for the output $\hat{y}(k)$ through the system considering 3 Volterra kernels expanded in the orthonormal Kautz basis. Figure 6 shows the linear, quadratic and cubic response estimated.

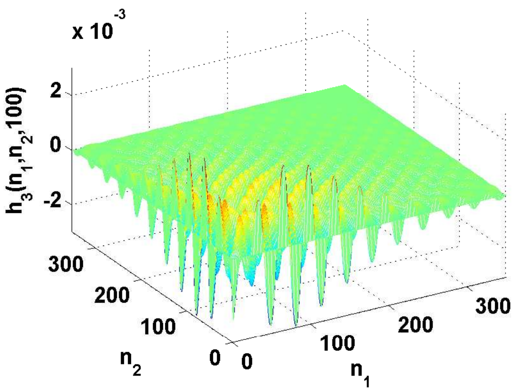

Figure 4: 3^{rd} Volterra kernel considering a surface cut in 100.

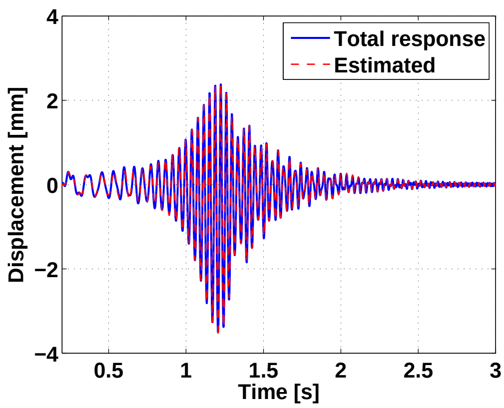

Figure 5: Comparison between the output measured $y(k)$ and total response $\hat{y}(k)$ using Volterra models.

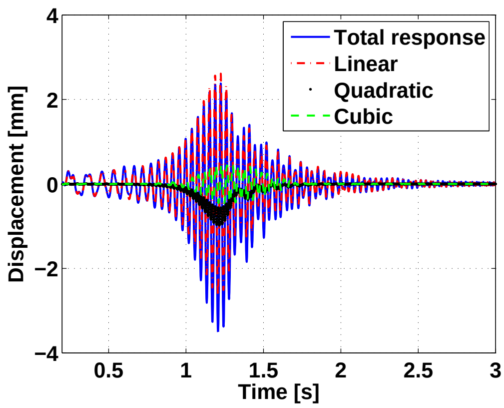

Figure 6: Contribution of each component in the total response.

4.2 Bilinear oscillator

The bilinear models are a special class of nonlinear polynomial models (Jácome et al., 1998). The model is described by:

$$m\ddot{y}(k) + c\dot{y}(k) + \eta\left(y(k)\right) = u(k) \qquad (12)$$

where $m = 1$ [kg], $c = 23.5619$ [N.s/m], $k_1 = 7.1\text{x}10^3$ [N/m], $k_2 = 2.84\text{x}10^4$ [N/m] and the nonlinear term $\eta(k) = k_1 y(k)$ if $y(k) < 0$ or $\eta(k) = k_2 y(k)$ if $y(k) \geq 0$. A chirp input signal $u(k)$, with amplitude 0.5 [N] sweeping the frequency range from 1 to 80 [Hz] was applied. A sampling frequency of 100 [Hz] and 1024 samples were used in this case. In the optimization results, it were obtained $J_1 = 10$ and $J_2 = J_3 = 6$ Kautz functions, $\omega_{n2} = 486.32$ and $\omega_{n3} = 502.65$ [rad/s], $\xi_2 = 0.107$ and $\xi_3 = 0.199$ considering 50 generations, the population size equals 200 and crossover fraction equals 0.8 in the genetic algorithm utilized. The Volterra kernels were estimated solving Eq. 8 and are illustrated, respectively, in the Figures 7, 8 and 9. It was estimated the output using the kernels identified and the Figure 10 the contribution of each response.

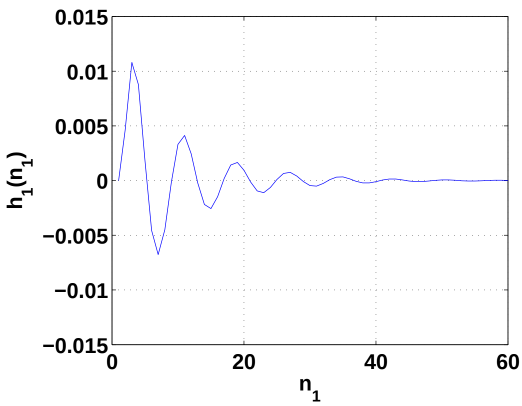

Figure 7: 1^{st} Volterra kernel.

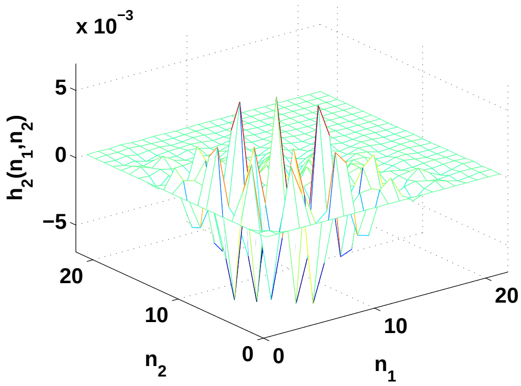

Figure 8: 2^{nd} Volterra kernel.

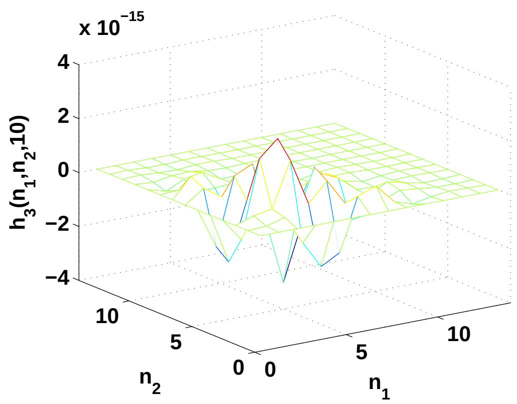

Figure 9: 3^{rd} Volterra kernel considering a surface cut in 10.

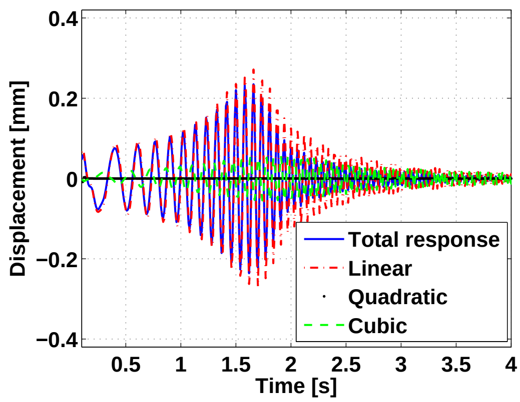

Figure 10: Contribution of each component in the total response.

4.3 Quadratically damped oscillator

This mechanical system is widely used in marine engineering, as in modeling oscillations of ships in balance (Gomes, 2011). The equation of motion for a quadratically damped oscillator, where the damping is proportional to the square of the velocity, is a non-linear second-order differential equation expressed by:

$$m\ddot{y}(k) + c\dot{y}(k)\left|\dot{y}(k)\right| + ky(k) = u(k) \tag{13}$$

where $m = 1$ [kg], $c = 19.5619$ [N.s^2/m^2], $k = 3.5\text{x}10^4$ [N/m]. A chirp input signal $u(k)$, with amplitude 0.5 [N] sweeping the frequency range from 1 to 70 [Hz] was generated in this case. A sampling frequency of 100 [Hz] and 1024 samples were used. In the optimization results, it were obtained $J_1 = J_3 = 8$ and $J_2 = 10$ Kautz functions, $\omega_{n2} = 389.96$ and $\omega_{n3} = 158.66$ [rad/s], $\xi_2 = 0.005$ and $\xi_3 = 0.0143$ considering 50 generations, the population size equals 200 and crossover fraction equals 0.8 in the genetic algorithm utilized. Figures 11, 12 and 13 show the Volterra kernels estimated.

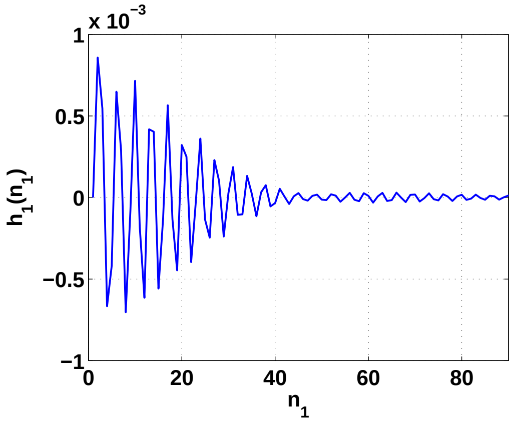

Figure 11: 1^{st} Volterra kernel.

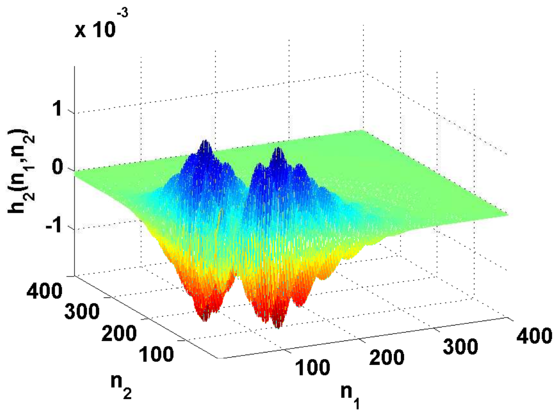

Figure 12: 2^{nd} Volterra kernel.

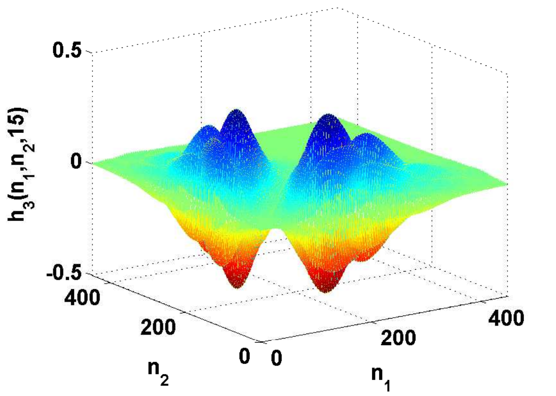

Figure 13: 3^{rd} Volterra kernel considering a surface cut in 15.

5 Final remarks

This paper proposed the application of Volterra models in benchmarks structures in order to obtain the dynamical information about the system, unknown a priori. By employing the Volterra formulation, it was possible to estimate considerable approximation for each response of the systems simulated. The numerical procedure based on genetic algorithm enabled to estimate the Volterra kernels using the orthonormal Kautz basis and drastically reduction on parameters to be estimated in the identification procedure. From the results shown, this approach can be useful in nonlinear mechanical systems for the purpose of to describe characteristics and dynamical information.

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